Quiz 10.1B

1. (a) $\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2 = 0.20 - 0.23 = -0.03$;

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{(p_1)(1-p_1)}{n_1} + \frac{(p_2)(1-p_2)}{n_2}} = \sqrt{\frac{(0.2)(0.8)}{240} + \frac{(0.23)(0.77)}{240}} = 0.037;$$

Since $n_1p_1 = 48$, $n_1(1-p_2) = 192$, $n_2p_2 = 55.2$, $n_2(1-p_2) = 184.8$, all of which are at least 10, the shape of the distribution is approximately Normal.

(b) $P(\hat{p}_1 - \hat{p}_2 > 0) = P\left(z > \frac{0-(-0.03)}{0.037}\right) = P(z > 0.81) = 0.2090$ **2.** (a) State: We wish to test $H_0: p_R - p_O = 0$; $H_a: p_R - p_O > 0$, where p_R and p_O are the proportion of red cars and other car, respectively, who are driving too fast. We will use a significance level of $\alpha = 0.05$.

Plan: The procedure is a two-sample z-test for the difference of proportions. Conditions: *Random:* The policemen chose cars randomly by rolling a die. *10%*: We can safely assume that the number of cars driving past the rest area is essentially infinite, so the 10% restriction does not apply. *Large counts:* The number of successes and failures in the two groups are 18, 10, 75, and 130—all of which are at least 10.

Do: $\hat{p}_R = \frac{18}{28} = 0.64$, $\hat{p}_O = \frac{75}{205} = 0.37$, and $\hat{p}_C = \frac{18+75}{28+205} = 0.40$, so

$$z = \frac{(0.64-0.37)-0}{\sqrt{\frac{(0.4)(0.6)}{28} + \frac{(0.4)(0.6)}{205}}} = \frac{0.27}{0.099} = 2.73;$$

One-tailed P-value = 1 – 0.9968 = 0.0032. [A 2-proportion z-test on the calculator yields $z = 2.807$ and a P-value of 0.0025] Conclude: A P-value of 0.0032 is less than $\alpha = 0.05$, so we reject H_0. We have sufficient evidence to conclude that the proportion of red cars that drive too fast on this highway is greater than the proportion of non-red cars that drive too fast.

(b) State: We wish to estimate, with 95% confidence, the difference $p_R - p_O$, as defined in part (a).

Plan: We should use a 2-sample z-interval for $p_M - p_F$. The conditions were addressed in part (a).

Do: The critical z for 95% confidence is 1.96, so the interval is

$$(0.64-0.37) \pm 1.96\left(\sqrt{\frac{(0.64)(0.36)}{28} + \frac{(0.37)(0.63)}{205}}\right) = 0.27 \pm 0.190\text{, or } (0.080,\ 0.460).$$

Conclude: We are 95% confident that the interval from 0.070 to 0.470 captures the true difference in the proportion of red cars and non-red cars that drive too fast on this highway.

Quiz 10.2A

1. (a) Let μ_1 = true mean pulse rate of patients similar to those in the experiment who take beta-blockers and μ_2 = mean pulse rate of similar patients who do not take beta blockers. Standard error of is $\sqrt{\frac{7.8^2}{30}+\frac{8.3^2}{30}} = 2.08$. (b) State: We wish to estimate, with 99% confidence, the difference $\mu_1 - \mu_2$, as defined in part (a). Plan: We should use a 2-sample *t*-interval for $\mu_1 - \mu_2$. Conditions: *Random:* The subjects were randomly assigned to the experimental treatments. *10%:* Since no sampling took place, the 10% condition does not apply. *Normal/Large Sample:* since both samples are at least 30, the Normal condition is satisfied. Do: Using the conservative degrees of freedom of 29, the critical *t*-value for 99% confidence is 2.756, so the interval is $(65.2-70.3) \pm 2.756\sqrt{\frac{7.8^2}{30}+\frac{8.3^2}{30}} = -5.10 \pm 5.73$, or $(-10.83, 0.63)$. [Using a calculator and 57.78 degrees of freedom, the interval is $(-10.64, 0.44)$]. Conclude: We are 99% confident that the interval from –10.83 to 0.63 captures the true difference in the mean pulse rate of patients receiving a beta-blocker during surgery and those who take a placebo.

(c) $H_0: \mu_1 - \mu_2 = 0$; $H_a: \mu_1 - \mu_2 < 0$. (d) Using Table A and df = 29, $0.01 < P\text{-value} < 0.02$. Using the calculator and df = 57.78, P-value = 0.0086. Since in both cases the P-value is less than $\alpha = 0.05$, we reject H_0. We have convincing evidence that the mean pulse rate of patients taking beta-blockers is lower than the mean pulse rate of patients taking a placebo.

Quiz 10.2B

1. (a) Let μ_T = mean amount of Tab-a-Cat Fern eats and μ_C = mean amount of Chow Lion she eats.

Standard error of $\overline{x}_T - \overline{x}_C$ is $\sqrt{\frac{3.45^2}{31}+\frac{4.62^2}{30}} = 1.047$. (b) State: We wish to estimate, with 99% confidence, the difference $\mu_T - \mu_C$, as defined in part (a). Plan: We should use a 2-sample *t*-interval for $\mu_1 - \mu_2$. Conditions: *Random:* Feedings of each type of food were randomly determined by coin flip. *10%:* We can view this study as randomly assigning 61 feedings to one of two groups, thus sampling did not take place, so the 10% condition does not apply. *Normal/Large Sample:* since both samples are at least 30, the Normal condition is satisfied. Do: Using the conservative degrees of freedom of 29, the critical *t*-value for 99% confidence is 2.756, so the interval is $(85.2 - 82.1) \pm 2.756\sqrt{\frac{3.45^2}{31}+\frac{4.62^2}{30}} = 3.10 \pm 2.88$, or $(0.22, 5.98)$. [Using a calculator and 53.64 degrees of freedom, the interval is $(0.30, 5.90)$]. Conclude: We are 99% confident that the interval from 0.22 to 5.98 captures the true difference in the mean amount of Tab-a-Cat Fern eats and the mean amount of Chow Lion she eats. (c) $H_0: \mu_T - \mu_C = 0$; $H_a: \mu_T - \mu_C > 0$. (d) Using Table A and df = 29, $0.0025 < P\text{-value} < 0.005$. Using the calculator and df = 53.64, *P*-value = 0.0023. Since in both cases the *P*-value is less than $\alpha = 0.01$, we reject H_0. We have convincing evidence that Fern eats more, on average, when offered Tab-a-Cat than when offered Chow Lion.