AP Calculus - §3.7a

Solve each optimization problem. For each problem, show a sketch, label your primary equation, and if necessary, label a secondary equation.

1) A farmer wants to construct a rectangular pigpen using 300 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

2) A rancher wants to construct two identical rectangular corrals using 100 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

3) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 6 greater than the negative number. What final product is the expert looking for?

4) A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

5) A company has started selling a new type of smartphone at the price of $\$140 - 0.1x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$70 and the labor and overhead for running the plant cost \$5000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

6) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 6. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

7) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 16$. What is the area of the largest rectangle that the student can draw?

8) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(2, 0)$?

9) Two vertical poles, one 15 ft high and the other 30 ft high, stand 24 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

10) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 20 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

Answers to AP Calculus - §3.7a

1) A = the area of the pigpen x = the length of the sides perpendicular to the stone wall
Function to maximize: $A = x(300 - 2x)$ where $0 < x < 150$
Dimensions of the pigpen: 75 ft (perpendicular to wall) by 150 ft (parallel to wall)

2) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral
Function to maximize: $A = 2x \cdot \dfrac{100 - 4x}{3}$ where $0 < x < 25$
Dimensions of each corall: $\dfrac{25}{2}$ ft (non-adjacent sides) by $\dfrac{50}{3}$ ft (adjacent sides)

3) P = the product of the two numbers x = the positive number
Function to minimize: $P = x(x - 6)$ where $-\infty < x < \infty$
Smallest product of the two numbers: -9

4) V = the volume of the box x = the length of the sides of the squares
Function to maximize: $V = (30 - 2x)(16 - 2x) \cdot x$ where $0 < x < 8$
Sides of the squares: $\dfrac{10}{3}$ in

5) p = the profit per day x = the number of items manufactured per day
Function to maximize: $p = x(140 - 0.1x) - (70x + 5000)$ where $0 \le x < \infty$
Optimal number of smartphones to manufacture per day: 350

6) A = the area of the rectangle x = half the base of the rectangle
Function to maximize: $A = 2x\sqrt{6^2 - x^2}$ where $0 < x < 6$
Area of largest rectangle: 36

7) A = the area of the rectangle x = half the base of the rectangle
Function to maximize: $A = 2x \cdot 2 \cdot \dfrac{\sqrt{16 - x^2}}{2}$ where $0 < x < 4$
Area of largest rectangle: 16

8) d = the distance from point $(2, 0)$ to a point on the curve x = the x-coordinate of a point on the curve
Function to minimize: $d = \sqrt{(x - 2)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$
Point on the curve that is closest to the point $(2, 0)$: $\left(\dfrac{3}{2}, \dfrac{\sqrt{6}}{2}\right)$

9) L = the total length of rope x = the horizontal distance from the short pole to the stake
Function to minimize: $L = \sqrt{x^2 + 15^2} + \sqrt{(24 - x)^2 + 30^2}$ where $0 \le x \le 24$
Stake should be placed: 8 ft from the short pole (or 16 ft from the long pole)

10) A = the area of the composite window x = the width of the bottom window = the diameter of the top window
Function to maximize: $A = x\left(\dfrac{20}{2} - \dfrac{x}{2} - \dfrac{\pi x}{4}\right) + \dfrac{1}{2}\pi \cdot \left(\dfrac{x}{2}\right)^2$ where $0 < x < \dfrac{80}{4 + \pi}$
Dimensions of the bottom window: $\dfrac{40}{4 + \pi}$ ft (width) by $\dfrac{20}{4 + \pi}$ ft (height)