

## Topic III Anticipating Patterns: Exploring Random Phenomena using Probability and Simulation

1. Probability is a measure of how likely an event is to occur. Match one of the probabilities that follow with each statement about an event.

0	0.01	0.3	0.6	0.99	1.00
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- The sun will rise in the west in the morning.
  - Thanksgiving will be on Thursday, November 22<sup>nd</sup> next year.
  - An event is very unlikely, but it will occur vary rarely.
  - The event will occur most of the time. Very rarely will it not occur.
  - Give an example of where the other 2 probabilities may occur.
2. What is the formula used for each of the following probabilities:

a. Addition Rule

b. Multiplication Rule

c. Conditional Probability

3. The type of medical care a patient receives may vary with the age of the patient. A large study of women who had a breast lump investigated whether or not each woman received a mammogram and a biopsy when the lump was discovered. Here are some probabilities estimated by the study. The entries in the table are the probabilities that both of two events occur; for example: 0.321 is the probability that a patient is under 65 years of age and the tests were done.

	Tests	Done
	Yes	No
Age Under 65	.321	.124
Age 65 and Over	.365	.190

- What is the probability that a patient in this study is under 65?
- Is 65 or over?
- What is the probability that the tests were done for a patient? That they were not done?
- Are the events  $A =$  (patient was 65 or older) and  $B =$  (the tests were done) independent? Were the tests omitted on older patients more or less frequently that would be the case if testing were independent of age?

4. Here are the counts (in thousands) of earned degrees in the United States in a recent year, classified by level and by the sex of the degree recipient:

	Bachelor's	Master's	Professional	Doctorate	Total
Female	616	194	30	16	
Male	529	171	44	26	
Total					

a. If you choose a degree

recipient at random, what is the probability that the person you choose is a woman?

- b. What is the conditional probability that you choose a woman, given that that person chosen received a professional degree?
- c. Are the events "choose a woman" and "choose a professional degree recipient" independent? How do you know?

5. Consolidated Builders has bid on two large construction projects. The company president believes that the probability of winning the first contract (event A) is 0.6, that the probability of winning the second (event B) is 0.4 and the joint probability of winning both jobs (event A and B) is 0.2.

- a. Draw the Venn diagram that illustrates the relationship between events A and B.

- b. Find the following probabilities:

$P(A \text{ or } B)$

$P(A \text{ and } B)$

$P(A, \text{ and Not } B)$

$P(\text{Not } A, \text{ and } B)$

$P(\text{not } A \text{ and not } B)$

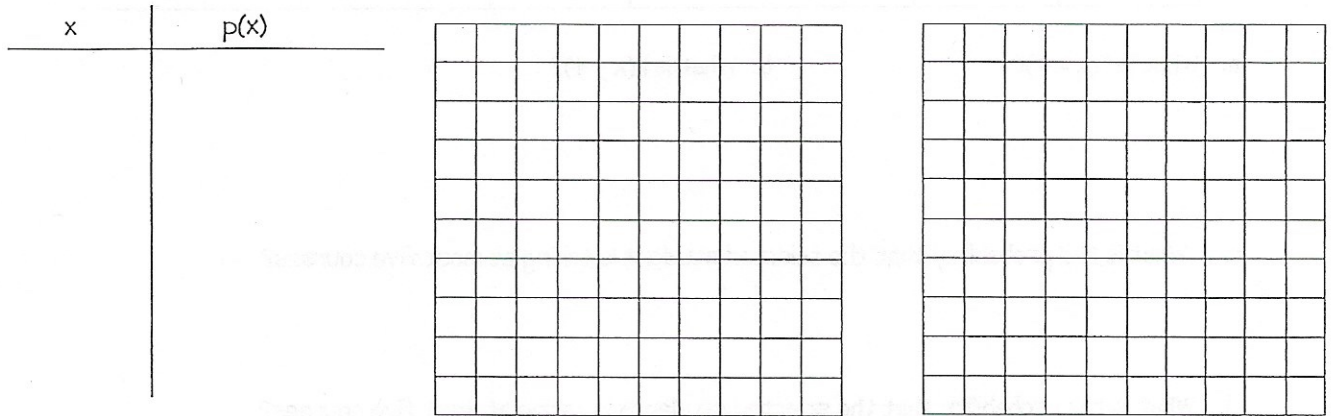
6. What is the difference between discrete and continuous random variables?



9. Among employed women, 25% have never been married. You select 10 employed women at random.

a. The number in your sample who have never been married has a binomial distribution. What are  $n$  and  $p$ ?

b. Create a binomial distribution table, a pdf histogram and a cdf histogram for this data.



c. What is the probability that exactly 2 of the 10 women in your sample have never been married?

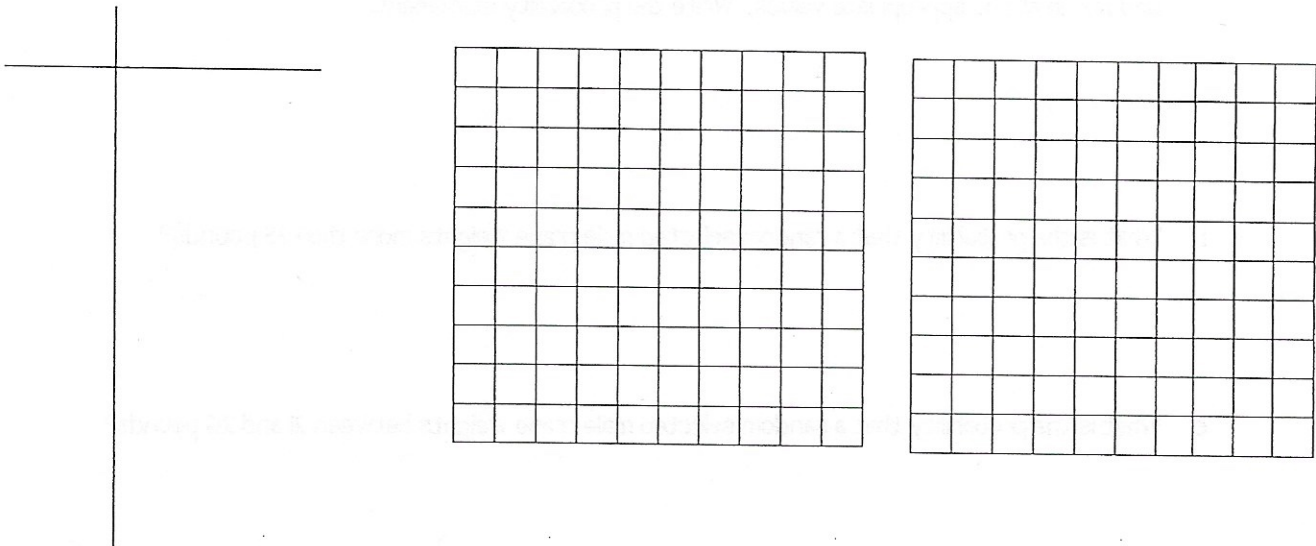
d. What is the probability that 2 or fewer have never been married?

e. What is the mean and standard deviation for this binomial distribution?

10. A basketball player makes 80% of his free throws. We put him on the free throw line and ask him to shoot free throws until he misses one. Let  $X$  = number of free throws the player takes until he misses.

a. What assumptions do you need to make in order for the geometric model to apply? With these assumptions, verify that  $X$  has a geometric distribution. What action constitutes "success" in this context?

b. Create a geometric distribution table for  $x$  values from 1 to 10. Create a pdf and a cdf.



c. What is the probability that the player will make 5 shots before he misses?

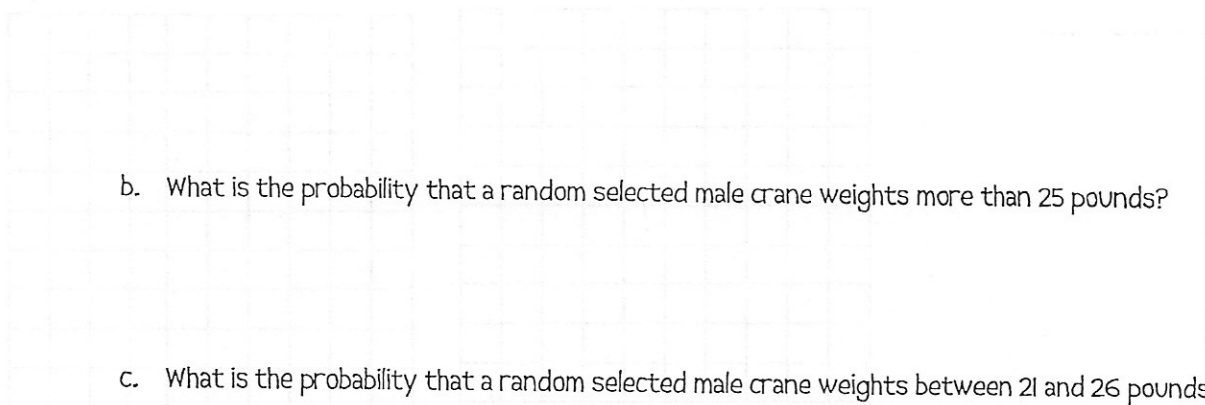
d. What is the probability that he will make at most 5 shots before he misses?

e. What is the mean of this geometric distribution?

11. The area under the curve for a normal distribution is represented by a bell-shaped curve.
- What are the properties of a normal distribution? Sketch a normal curve.

12. A certain population of whooping cranes that migrate between Wisconsin and Florida every year has a SRS taken. The sample of 15 male cranes were weighed before they left Wisconsin to begin their trip. The mean weight of the 15 males was found to be 22.7 pounds with a standard deviation of 2.3 pounds. Why is this population considered a normal distribution?

- What is the probability that a random selected male crane weights less than 20 pounds? Sketch the curve and put in all the appropriate values. Write the probability statement.



- What is the probability that a random selected male crane weights more than 25 pounds?

- What is the probability that a random selected male crane weights between 21 and 26 pounds?

- When these cranes reach Florida, another random sample of 25 male cranes is weighted and measured. The mean weight is recorded at 19.5 pounds with a standard deviation of 1.7 pounds. Using this sample statistics, make a prediction about another sample of 25 from the same population, what is the probability that the mean of the samples will be between 15 and 22 pounds?

- What is the probability that the sampling distribution of 25 cranes would have a mean greater than 23 pounds?

- What is the probability that the sampling distribution would be less than 18 pounds?

13. The Helsinki Heart Study asks whether the anti-cholesterol drug gemfibrozil will reduce heart attacks. In planning such an experiment, the researchers must be confident that the sample sizes are large enough to enable them to observe enough heart attacks. The Helsinki study plans to give gemfibrozil to 2000 men and a placebo to another 2000 men. The probability of a heart attack during the 5-year period of the study for men this age is about 0.04. We can think of the study participants as an SRS from a large population, of which the proportion  $p = 0.04$  will have heart attacks.
- What is the mean number of heart attacks that the study will find in one group of 2000 men if the treatment doesn't change the probability of 0.04?
  - What is the probability that the group will suffer at least 75 heart attacks? Sketch the curve, show all the work and write the probability statement.
14. Children in kindergarten are sometimes given the Raven Progressive Matrices Test (RPMT) to assess their readiness for learning. Experience at Southward Elementary School suggests that the RPMT scores for its kindergarten pupils have a mean of 13.6 with a standard deviation of 3.1. The distribution is close to normal. Mr. Brown has 22 children in his kindergarten class this year.
- What is the probability that class's mean score will be less than 12.0?
  - Mr. Brown suspects that the class RPMT scores will be unusually low because the test was interrupted by a fire drill. He wants to find the level  $L$  such that there is only a probability of 0.05 that the mean score of his class fall below  $L$ . What is this value of  $L$ . (Hint: this requires you to find the  $z$ -score and then convert to the  $x$ -score.)
15. Explain what is meant by the Law of Large Numbers. How does this law apply to sampling distributions?
16. What is the Central Limit Theorem? How is the CLT used in sampling distributions?