Summary of Key Concepts and Formulas

Term or Formula	Comment
Statistic	Any quantity whose value is computed from sample data.
Sampling distribution	The probability distribution of a statistic: The sampling distribution describes the long-run behavior of the statistic.
Sampling distribution of $\bar{x}$	The probability distribution of the sample mean $\bar{x}$, based on a random sample of size n. Properties of the $\bar{x}$ sampling distribution: $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ (where μ and σ are the population mean and standard deviation, respectively). In addition, when the population distribution is normal or the sample size is large, the sampling distribution of $\bar{x}$ is (approximately) normal.
Central Limit Theorem	This important theorem states that when n is sufficiently large, the $\bar{x}$ distribution will be approximately normal. The standard rule of thumb is that the theorem can safely be applied when n exceeds 30.
Sampling distribution of $\hat{p}$	The probability distribution of the sample proportion $\hat{p}$, based on a random sample of size n. When the sample size is sufficiently large, the sampling distribution of $\hat{p}$ is approximately normal, with $\mu_{\hat{p}} = p$ and $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$ (where p is the true population proportion).

Supplementary Exercises 8.34 – 8.40

8.34 The nicotine content in a single cigarette of a particular brand has a distribution with mean .8 mg and standard deviation .1 mg. If 100 of these cigarettes are analyzed, what is the probability that the resulting sample mean nicotine content will be less than .79? Less than .77?

8.35 Let $x_1, x_2, \ldots, x_{100}$ denote the actual net weights (in lb) of 100 randomly selected bags of fertilizer. Suppose that the weight of a randomly selected bag has a distribution with mean 50 lb and variance 1 lb^2. Let $\bar{x}$ be the sample mean weight ($n = 100$).

a. Describe the sampling distribution of $\bar{x}$.

b. What is the probability that the sample mean is between 49.75 lb and 50.25 lb?

c. What is the probability that the sample mean is less than 50 lb?

8.36 Suppose that 20% of the subscribers of a cable television company watch the shopping channel at least once a week. The cable company is trying to decide whether to replace this channel with a new local station. A survey of 100 subscribers will be undertaken. The cable company has decided to keep the shopping channel if the sample proportion is greater than .25. What is the approximate probability that the cable company will keep the shopping channel, even though the true proportion who watch it is only .20?

8.37 Although a lecture period at a certain university lasts exactly 50 minutes, the actual lecture time of a statistics instructor on any particular day has a distribution with mean value 52 min and standard deviation 2 min. Suppose that times of different lectures are independent of one another. Let $\bar{x}$ represent the mean of 36 randomly selected lecture times.

a. What are the mean value and standard deviation of the sampling distribution of $\bar{x}$?

b. What is the probability that the sample mean exceeds 50 min? 55 min?

8.38 Water permeability of concrete is an important characteristic in assessing suitability for various applications. Permeability can be measured by letting water flow across the surface and determining the amount lost (in/hr). Suppose that the permeability index x for a randomly selected concrete specimen of a particular type is normally distributed with mean value 1000 and standard deviation 150.

a. How likely is it that a single specimen will have a permeability index between 850 and 1300?

b. If the permeability index is determined for each specimen in a random sample of size 10, how likely is it that the sample average permeability index will be between 950 and 1100? Between 850 and 1300?

8.39 *Newsweek* (Nov. 23, 1992) reported that 40% of all U.S. employees participate in "self-insurance" health plans ($p = .40$).

a. In a random sample of 100 employees, what is the approximate probability that at least half of those in the sample participate in such a plan?

b. Suppose you were told that at least 60 of the 100 employees in a sample from your state participated in such a plan. Would you think $p = .40$ for your state?