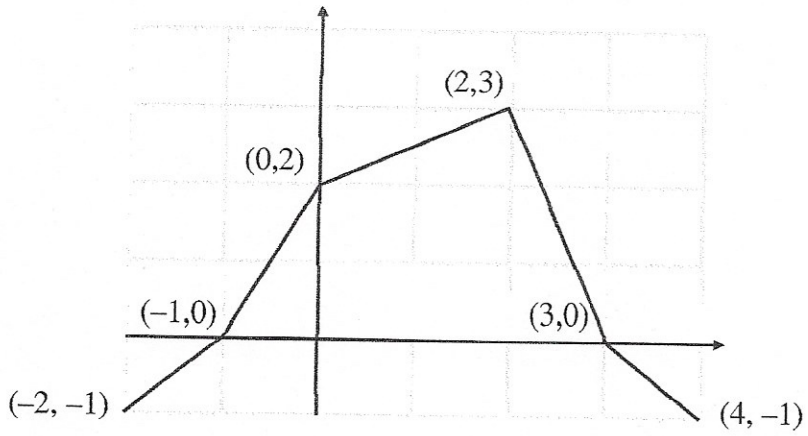
The Detective's Hat Function



The graph of the function f shown above is a piecewise continuous function defined on $[-2, 4]$. The graph of f consists of five line segments.

Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

1. Find each of the following.

   (a) $g(-2)$ (b) $g(-1)$ (c) $g(0)$ (d) $g(2)$ (e) $g(3)$ (f) $g(4)$

2. Explain the procedure you followed to answer question 1.

3. Find each of the following.

   (a) $g'(-1)$ (b) $g'(0)$ (c) $g'(2)$ (d) $g'(4)$

4. Explain the procedure you followed to answer question 3.

5. Explain why g must be a continuous function on $[-2, 4]$.

6. Write the equation for $g'(x)$ on the interval $[0, 2]$.

7. Write the equation for the line tangent to g at $x = 1$. Justify your answer.

8. Does $g''(0)$ exist? Explain your reasoning.

9. Will a point of inflection for g exist when $x = 0$? Explain your reasoning.

10. For what values of x in the open interval $(-2, 4)$ is g increasing? Explain your reasoning.

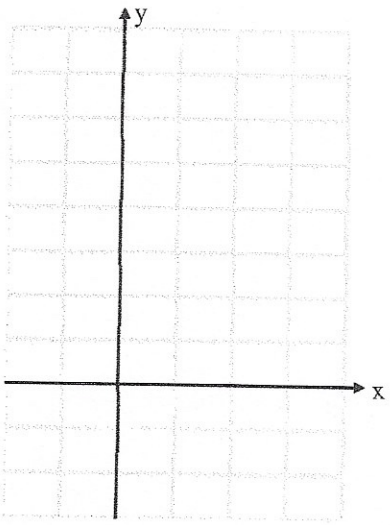
11. For what values of x in the open interval $(-2, 4)$ is g decreasing?

12. For what values of x in the open interval $(-2, 4)$ is g concave up? Explain your reasoning.

13. For what values of x in the open interval $(-2, 4)$ is g concave down?

14. Find the maximum and the minimum values of g on the closed interval $[-2, 4]$. Justify your answers.

15. On the axes provided, sketch the graph of function g on the closed interval $[-2, 4]$.



For questions 15 – 17, let h be the function given by $h(x) = \int_{-2}^{x} f(t)\,dt$.

16. Find each of the following.

(a) $h(-2)$ (b) $h(-1)$ (c) $h(0)$ (d) $h(2)$ (e) $h(3)$ (f) $h(4)$

17. Find the following and explain your reasoning.

(a) $h'(-1)$ (b) $h'(0)$ (c) $h'(2)$ (d) $h'(4)$

18. $g(x) - h(x) = k$, where k is a constant. Find the value of k and explain your reasoning.

19. If $w(x) = \int_{3}^{x} f(t)\,dt$, find $w(0)$.

20. $w(x)$ can also be defined as $w(x) = r + \int_{0}^{x} f(t)\,dt$ where r is a constant. What is the value of r?