Evaluating the integrals, we get:

$$\frac{y^2}{2}=\frac{x^4}{4}+x+C$$

Next, we Plug In $y=2$ and $x=1$ to solve for C. We get: $2=\frac{1}{4}+1+C$ and so $C=\frac{3}{4}$. This gives us:

$$\frac{y^2}{2}=\frac{x^4}{4}+x+\frac{3}{4}$$

Now, if we substitute $x=2$, we get:

$$\frac{y^2}{2}=4+2+\frac{3}{4}=\frac{27}{4}$$

Solving for y, we get:

$$y=\pm\sqrt{\frac{27}{2}}$$

The answer is (E).

Problem 29. The graph of $y=5x^4-x^5$ has an inflection point (or points) at

In order to find the inflection point(s) of a polynomial, we need to find the values of x where its second derivative is zero.

First, we find the second derivative.

$$\frac{dy}{dx}=20x^3-5x^4$$

$$\frac{d^2y}{dx^2}=60x^2-20x^3$$

Now, let's set the second derivative equal to zero and solve for x.

$$60x^2-20x^3=0$$

$$20x^2(3-x)=0$$

$$x=3$$

This is the point of inflection. $x=0$ is not a point of inflection because $\frac{d^2y}{dx^2}$ does not change sign there. If you are unsure that these are correct, graph the function with a calculator and look at the picture.

The answer is (B).

Problem 30. The average value of $f(x)=e^{4x^2}$ on the interval $\left[-\frac{1}{4},\frac{1}{4}\right]$ is

In order to find the average value, we use the Mean Value Theorem for Integrals, which says that the average value of $f(x)$ on the interval $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

Here, we have

$$\frac{1}{\frac{1}{4}+\frac{1}{4}}\int_{-\frac{1}{4}}^{\frac{1}{4}} e^{4x^2}\,dx = 2\int_{-\frac{1}{4}}^{\frac{1}{4}} e^{4x^2}\,dx$$

You can't evaluate this integral using any of the techniques that you have studied so far, so use the calculator to evaluate the integral numerically.

Remember this: Any integral on the AP that contains an e^{x^2} term that is not multiplied by x, must be integrated using the calculator.

You should get approximately 1.09 . **The AP always expects you to round to three decimal places.**

The answer is (C).

Problem 31. $\int_0^1 \tan x\,dx =$

First, rewrite the integral as $\int_0^1 \frac{\sin x}{\cos x}\,dx$.

Now, we can use u-substitution to evaluate the integral. Let $u=\cos x$. Then $du=-\sin x$. We can also change the limits of integration. The lower limit becomes $\cos 0=1$ and the upper limit becomes $\cos 1$, which we leave alone. Now we perform the substitution and we get:

$$-\int_1^{\cos 1}\frac{du}{u}$$

Evaluating the integral, we get: $-\ln u\Big|_1^{\cos 1}=-\ln(\cos 1)+\ln 1=-\ln(\cos 1)$. This log is also equal to $\ln(\sec 1)$.

The answer is (D).

Problem 32. $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \sin^2 t \, dt =$

The Second Fundamental Theorem of Calculus tells us how to find the derivative of an integral. It says that $\dfrac{d}{dx}\displaystyle\int_c^u f(t)\,dt = f(u)\dfrac{du}{dx}$, where c is a constant and u is a function of x.

Here we can use the theorem to get:

$$\frac{d}{dx}\int_0^{x^2} \sin^2 t \, dt = \left(\sin^2\left(x^2\right)\right)(2x) \text{ or } 2x\sin^2\left(x^2\right)$$

The answer is (B).

Problem 33. Find the value(s) of $\dfrac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 2$.

Here, we use implicit differentiation to find $\dfrac{dy}{dx}$:

$$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

Now we plug $y = 1$ into the original equation to find its corresponding x values.

$$x^2 + 1 = 5$$

$$x^2 = 4$$

$$x = \pm 2$$

Now Plug In the x and y values to find the value of $\dfrac{dy}{dx}$.

For $y = 1$ and $x = 2$, we get:

$$2(2)(1) + (2)^2\frac{dy}{dx} + 2(1)\frac{dy}{dx} = 0$$

Solving for $\dfrac{dy}{dx}$, we get:

$$4 + 6\frac{dy}{dx} = 0 \text{ and } \frac{dy}{dx} = -\frac{2}{3}$$

For $y = 1$ and $x = -2$, we get:

$$2(-2)(1) + (-2)^2\frac{dy}{dx} + 2(1)\frac{dy}{dx} = 0$$

Solving for $\frac{dy}{dx}$, we get:

$$-4+6\frac{dy}{dx}=0 \text{ and } \frac{dy}{dx}=\frac{2}{3}$$

The answer is (D).

Problem 34. The graph of $y=x^3-2x^2-5x+2$ has a local maximum at

First, let's find $\frac{dy}{dx}$:

$$\frac{dy}{dx}=3x^2-4x-5$$

Next, set the derivative equal to zero and solve for x.

$$3x^2-4x-5=0$$

Using the quadratic formula (or your calculator), we get:

$$x=\frac{4\pm\sqrt{16+60}}{6}\approx 2.120, -0.786$$

Let's use the second derivative test to determine which is the maximum. We take the second derivative and then Plug In the critical values that we found when we set the first derivative equal to zero. **If the sign of the second derivative at a critical value is positive, then the curve has a local minimum there. If the sign of the second derivative is negative, then the curve has a local maximum there.**

The second derivative is: $\frac{d^2y}{dx^2}=6x-4$. This is negative at $x=-0.786$, so the curve has a local maximum there. Now we plug $x=-0.786$ into the original equation to find the y-coordinate of the maximum. We get approximately 4.209. Therefore, the curve has a local maximum at $(-0.786, 4.209)$.

The answer is (D).

Problem 35. Approximate $\int_0^1 \sin^2 x\, dx$ using the trapezoid rule with n = 4, to three decimal places.

The Trapezoid Rule enables us to approximate the area under a curve with a fair degree of accuracy. The rule says that the area between the x-axis and the curve $y=f(x)$, on the interval $[a,b]$, with n trapezoids, is:

$$\frac{1}{2}\frac{b-a}{n}\left[y_0+2y_1+2y_2+2y_3+\ldots+2y_{n-1}+y_n\right]$$

Using the rule here, with $n=4$, $a=0$, and $b=1$, we get:

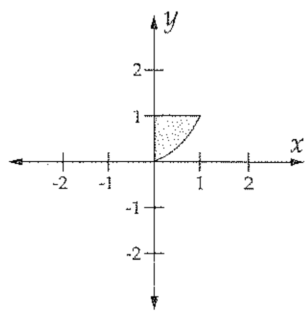
$$\frac{1}{2}\cdot\frac{1}{4}\left[\sin^2 0+2\sin^2\frac{1}{4}+2\sin^2\frac{1}{2}+2\sin^2\frac{3}{4}+\sin^2 1\right]$$

This is approximately 0.277.

The answer is (A).

Problem 36. The volume generated by revolving about the x-axis the region above the curve $y=x^3$, below the line $y=1$, and between $x=0$ and $x=1$ is

First, make a quick sketch of the region.



We can find the volume by taking a vertical slice of the region. The formula for the volume of a solid of revolution around the x-axis, using a vertical slice bounded from above by the curve $f(x)$ and from below by $g(x)$, on the interval $[a,b]$, is:

$$\pi\int_a^b [f(x)]^2-[g(x)]^2\,dx$$

Here, we get:

$$\pi\int_0^1 (1)^2-(x^3)^2\;dx$$

Now we have to evaluate the integral. First, expand the integrand to get:

$$\pi\int_0^1 (1-x^6)dx$$

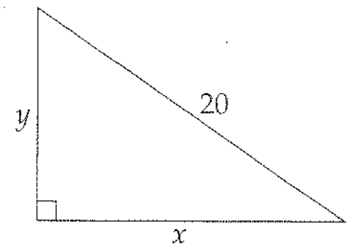
Next, integrate to get:

$$\pi\left(x-\frac{x^7}{7}\right)\Bigg|_0^1=\pi\left(1-\frac{1}{7}\right)=\frac{6\pi}{7}$$

The answer is (E).

PROBLEM 37. A 20 foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor? (in ft/sec.)

First, let's make a sketch of the situation:



We are given that $\frac{dy}{dt} = -5$ (it's negative because the ladder is sliding down and its customary to make the upward direction positive), and we want to find $\frac{dx}{dt}$ when $y = 10$.

We can find a relationship between x and y using the Pythagorean Theorem. We get: $x^2 + y^2 = 400$.

Now, taking the derivative with respect to t, we get:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0, \text{ which can be simplified to } x\frac{dx}{dt} = -y\frac{dy}{dt}$$

Next, we need to find x when $y = 10$.

Using the Pythagorean Theorem,

$$x^2 + 10^2 = 400, \text{ so } x = \sqrt{300} \approx 17.321$$

Now, Plug Into the equation above to get:

$$17.321\frac{dx}{dt} = -10(-5) \text{ and } \frac{dx}{dt} \approx 2.887$$

The answer is (B).

PROBLEM 38. $\int \frac{\ln x}{3x}\,dx =$

We can evaluate the integral with u-substitution.

Let $u = \ln x$. Then $du = \frac{dx}{x}$.

Substituting, we get: $\frac{1}{3}\int u\,du$.

Now, we can evaluate the integral: $\frac{1}{3}\frac{u^2}{2} + C$.

Substituting back, we get: $\frac{\ln^2|x|}{6} + C$.

The answer is (D).

PROBLEM 39. Find two nonnegative numbers x and y whose sum is 100 and for which x^2y is a maximum.

Let's set $P = x^2y$. We want to maximize P, so we need to eliminate one of the variables. We are also given that $x + y = 100$, so we can solve this for y and substitute. $y = 100 - x$, so $P = x^2(100 - x) = 100x^2 - x^3$.

Now we can take the derivative.

$$\frac{dP}{dx} = 200x - 3x^2$$

Set the derivative equal to zero and solve for x.

$$200x - 3x^2 = 0$$

$$x(200 - 3x) = 0$$

$$x = 0 \text{ or } x = \frac{200}{3} \approx 66.667$$

Now we can use the second derivative to find the maximum. $\frac{d^2P}{dx^2} = 200 - 6x$.

If we Plug In $x = 66.667$, the second derivative is negative, so P is a maximum at $x = 66.667$. Solving for y, we get $y \approx 33.333$.

The answer is (E).

Problem 40. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

The function $t + \ln t$ is always positive on the interval, so we can find the distance traveled by evaluating the integral

$$\int_1^5 (t + \ln t)\, dt$$

We can evaluate the integral numerically using the calculator.

You should get approximately 16.047 . **The AP always expects you to round to three decimal places.**

The answer is (C).

Problem 41. $\int \sin^5(2x)\cos(2x)\, dx =$

We can evaluate this integral using u-substitution.

Let $u = \sin(2x)$. Then $du = 2\cos(2x)dx$, which we can rewrite as $\frac{1}{2}du = \cos(2x)dx$.

Substituting into the integrand, we get:

$$\frac{1}{2}\int u^5 du$$

Evaluating the integral gives us:

$$\frac{1}{2}\frac{u^6}{6} + C = \frac{u^6}{12} + C$$

Substituting back, we get:

$$\frac{\sin^6(2x)}{12} + C$$

The answer is (A).

Problem 42. The volume of a cube is increasing at a rate proportional to its volume at any time t. If the volume is 8 ft^3 originally, and 12 ft^3 after 5 seconds, what is its volume at $t = 12$ seconds?

When we see a phrase where something is increasing at a rate "proportional to itself at any time t", this means that we set up the differential equation

$$\frac{dV}{dt} = kV$$

(or whatever the appropriate variable is)

We solve this differential equation using separation of variables.

First, move the V to the left side and the dt to the right side, to get:

$$\frac{dV}{V} = kdt$$

Now, integrate both sides:

$$\int \frac{dV}{V} = k\int dt$$

$$\ln V = kt + C$$

Next, it's traditional to put the equation in terms of V. We do this by exponentiating both sides to the base e. We get:

$$V = e^{kt+C}$$

Using the rules of exponents, we can rewrite this as:

$$V = e^{kt}e^{C}$$

Finally, because e^{C} is a constant, we can rewrite the equation as:

$$V = Ce^{kt}$$

Now, we use the initial condition that $V = 8$ at time $t = 0$ to solve for C.

$$8 = Ce^{0} = C(1) = C$$

This gives us

$$V = 8e^{kt}$$

Next, we use the condition that $V = 12$ at time $t = 5$ to solve for k.

$$12 = 8e^{5k}$$

$$\frac{3}{2} = e^{5k}$$

$$\ln\frac{3}{2} = 5k$$

$$k = \frac{1}{5}\ln\frac{3}{2}$$

This gives us:

$$V = 8e^{\left(\frac{1}{5}\ln\frac{3}{2}\right)t}$$

Finally, we Plug In $t = 12$ and solve for V:

$$V = 8e^{\left(\frac{1}{5}\ln\frac{3}{2}\right)(12)} \approx 21.169$$

The answer is (A).

Problem 43. If $f(x) = \left(1 + \frac{x}{20}\right)^5$, find $f''(40)$.

The first derivative is:

$$f'(x) = 5\left(1 + \frac{x}{20}\right)^4\left(\frac{1}{20}\right) = \frac{1}{4}\left(1 + \frac{x}{20}\right)^4$$

The second derivative is:

$$f''(x) = \frac{1}{4} \cdot 4\left(1 + \frac{x}{20}\right)^3\left(\frac{1}{20}\right) = \frac{1}{20}\left(1 + \frac{x}{20}\right)^3$$

Evaluating this at $x = 40$, we get:

$$f''(x) = \frac{1}{20}\left(1 + \frac{40}{20}\right)^3 = \frac{27}{20} = 1.350$$

The answer is (B).

Problem 44. A particle's height at a time $t \geq 0$ is given by $h(t) = 100t - 16t^2$. What is its maximum height?

First, let's take the derivative: $h'(t) = 100 - 32t$

Now, we set it equal to zero and solve for t: $100 - 32t = 0$

$$t = \frac{100}{32}$$

Now, to solve for the maximum height, we simply plug $t = \frac{100}{32}$ back into the original equation for height:

$$h\left(\frac{100}{32}\right) = 100\left(\frac{100}{32}\right) - 16\left(\frac{100}{32}\right)^2 = 156.250$$

By the way, we know that this is a maximum not a minimum because the second derivative is -32, which means that the critical value will give us a maximum not a minimum.

The answer is (B).

PROBLEM 45. If $f(x)$ is continuous and differentiable and

$$f(x)=\begin{cases} ax^4+5x\ ; x\le 2 \\ bx^2-3x\ ; x>2 \end{cases},\ \text{then } b=$$

In order to solve this for b, we need $f(x)$ to be differentiable at $x=2$.

If we plug $x=2$ into both pieces of this piecewise function, we get:

$$f(x)=\begin{cases} 16a+10; x\le 2 \\ 4b-6; \quad x>2 \end{cases},$$

so we need $16a+10=4b-6$.

Now, if we take the derivative of both pieces of this function and Plug In $x=2$ we get:

$$f'(x)=\begin{cases} 32a+5; x\le 2 \\ 4b-3; \quad x>2 \end{cases},\ \text{so we need } 32a+5=4b-3$$

Solving the simultaneous equations, we get $a=\frac{1}{2}$ and $b=6$.

The answer is (D).