CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—40

Percent of total grade—50

Part A consists of 28 questions that will be answered on side 1 of the answer sheet. Following are the directions for Section I, Part A)

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $g(x) = \frac{1}{32}x^4 - 5x^2$, find $g'(4)$.

   (A) –72
   (B) –32
   (C) –24
   (D) 24
   (E) 32

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2. The domain of the function $f(x) = \sqrt{4 - x^2}$ is

   (A) $x < -2$ or $x > 2$
   (B) $x \leq -2$ or $x \geq 2$
   (C) $-2 < x < 2$
   (D) $-2 \leq x \leq 2$
   (E) $x \leq 2$

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3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

(A) 0
(B) 10
(C) −10
(D) 5
(E) The limit does not exist

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4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$.

(A) $\frac{(5x^4 - 1)}{(3x^2)}$

(B) $\frac{(5x^4 - 1) - (3x^2)}{(x^3 + 7)}$

(C) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)}$

(D) $\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$

(E) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$

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5. Evaluate $\lim_{h \to 0} \frac{5\left(\frac{1}{2}+h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

(A) $\frac{5}{2}$

(B) $\frac{5}{16}$

(C) 40

(D) 160

(E) The limit does not exist.

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6. $\int x\sqrt{3x}\, dx =$

(A) $\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$

(B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$

(C) $\frac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$

(D) $2\sqrt{3x} + C$

(E) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$

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7. Find k so that $f(x)=\begin{cases}\dfrac{x^2-16}{x-4} & ;x\neq 4\\ k & ;x=4\end{cases}$ is continuous for all x.

(A) All real values of k make $f(x)$ continuous for all x.

(B) 0

(C) 16

(D) 8

(E) There is no real value of k that makes $f(x)$ continuous for all x.

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8. Which of the following integrals correctly gives the area of the region consisting of all points above the x-axis and below the curve $y=8+2x-x^2$?

(A) $\int_{-2}^{4}\left(x^2-2x-8\right)dx$

(B) $\int_{-4}^{2}\left(8+2x-x^2\right)dx$

(C) $\int_{-2}^{4}\left(8+2x-x^2\right)dx$

(D) $\int_{-4}^{2}\left(x^2-2x-8\right)dx$

(E) $\int_{2}^{4}\left(8+2x-x^2\right)dx$

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9. If $f(x)=x^2\cos 2x$, find $f'(x)$.

(A) $2x\sin 2x$

(B) $-2x\cos 2x+2x^2\sin 2x$

(C) $-4x\sin 2x$

(D) $2x\cos 2x-2x^2\sin 2x$

(E) $2x-2\sin 2x$

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10. An equation of the line tangent to $y = 4x^3 - 7x^2$ at $x = 3$ is

(A) $y + 45 = 66(x + 3)$

(B) $y - 45 = 66(x - 3)$

(C) $y = 66x$

(D) $y = 66(x - 3)$

(E) $y - 45 = \frac{-1}{66}(x - 3)$

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11. $\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $-\frac{2\pi}{3}$

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12. Find a positive value c, for x, that satisfies the conclusion of the Mean Value Theorem for Derivatives for $f(x) = 3x^2 - 5x + 1$ on the interval $[2,5]$.

(A) 1

(B) $\frac{13}{6}$

(C) $\frac{11}{6}$

(D) $\frac{23}{6}$

(E) $\frac{7}{2}$

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13. Given $f(x) = 2x^2 - 7x - 10$, find the absolute maximum of $f(x)$, on $[-1,3]$.

(A) -1

(B) $\frac{7}{4}$

(C) -13

(D) $-\frac{129}{8}$

(E) 0

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14. Find $\frac{dy}{dx}$ if $x^3y + xy^3 = -10$.

(A) $\left(3x^2 + 3xy^2\right)$

(B) $-\left(3x^2 + 3xy^2\right)$

(C) $\frac{\left(3x^2y + y^3\right)}{\left(3xy^2 + x^3\right)}$

(D) $-\frac{\left(3x^2y + y^3\right)}{\left(3xy^2 + x^3\right)}$

(E) $-\frac{\left(x^2y + y^3\right)}{\left(xy^2 + x^3\right)}$

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15. If $f(x)=\sqrt{1+\sqrt{x}}$, find $f'(x)$.

(A) $\dfrac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(B) $\dfrac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

(C) $\dfrac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(E) $\dfrac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

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16. $\int 7xe^{3x^2}dx =$

(A) $\dfrac{1}{42}e^{3x^2}+C$

(B) $\dfrac{6}{7}e^{3x^2}+C$

(C) $\dfrac{7}{6}e^{3x^2}+C$

(D) $7e^{3x^2}+C$

(E) $42e^{3x^2}+C$

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17. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ through $(2,-1)$.

(A) $-9x + 8y - 26 = 0$

(B) $9x - 8y - 26 = 0$

(C) $9x - 8y - 106 = 0$

(D) $8x + 9y - 17 = 0$

(E) $9x + 16y - 2 = 0$

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18. A particle's position is given by $s = t^3 - 6t^2 + 9t$. What is its acceleration at time $t = 4$?

(A) 0

(B) 9

(C) –9

(D) –12

(E) 12

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19. If $f(x) = 3^{\pi x}$, then $f'(x) =$

(A) $\dfrac{3^{\pi x}}{\pi \ln 3}$

(B) $\dfrac{3^{\pi x}}{\ln 3}$

(C) $\dfrac{3^{\pi x}}{\pi}$

(D) $\pi \cdot 3^{\pi x - 1}$

(E) $\pi \ln 3 \cdot 3^{\pi x}$

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20. The average value of $f(x)=\frac{1}{x}$ from $x = 1$ to $x = e$ is

(A) $\frac{1}{e+1}$

(B) $\frac{1}{1-e}$

(C) $e-1$

(D) $1-\frac{1}{e^2}$

(E) $\frac{1}{e-1}$

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21. If $f(x)=\sin^2 x$, find $f'''(x)$.

(A) $-\sin^2 x$

(B) $2\cos 2x$

(C) $\cos 2x$

(D) $-4\sin 2x$

(E) $-\sin 2x$

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22. Find the slope of the normal line to $y = x + \cos xy$ at $(0,1)$.

(A) 1
(B) -1
(C) 0
(D) 2
(E) Undefined

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23. $\int e^x \cdot e^{3x} dx =$

(A) $\frac{1}{3}e^{3x} + C$

(B) $\frac{1}{4}e^{4x} + C$

(C) $\frac{1}{4}e^{5x} + C$

(D) $4e^{4x} + C$

(E) $4e^{5x} + C$

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24. $\lim_{x \to 0} \frac{\tan^3(2x)}{x^3} =$

(A) -8
(B) -2
(C) 2
(D) 8
(E) The limit does not exist.

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25. A solid is generated when the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi\int_0^1 \left(1+\sin^4 x\right) dx$

(B) $\pi\int_0^1 \left(1+\sin^2 x\right)^2 dx$

(C) $\pi\int_0^{\frac{\pi}{2}} \left(1+\sin^4 x\right) dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \left(1+\sin^2 x\right)^2 dx$

(E) $\pi\int_0^{\frac{\pi}{2}} \left(1+\sin^2 x\right) dx$

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26. If $y = \left(\frac{x^3-2}{2x^5-1}\right)^4$, find $\frac{dy}{dx}$ at $x = 1$.

(A) –52 (B) –28 (C) –13 (D) 13 (E) 52

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27. $\int x\sqrt{5-x}\,dx =$

(A) $-\frac{10}{3}(5-x)^{\frac{3}{2}}$

(B) $\sqrt{\frac{5x^2}{2}-\frac{x^3}{3}}+C$

(C) $\frac{10}{3}\sqrt{\frac{5x^2}{2}-\frac{x^3}{3}}+C$

(D) $10(5-x)^{\frac{1}{2}}+\frac{2}{3}(5-x)^{\frac{3}{2}}+C$

(E) $-\frac{10}{3}(5-x)^{\frac{3}{2}}+\frac{2}{5}(5-x)^{\frac{5}{2}}+C$

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28. If $\frac{dy}{dx}=\frac{x^3+1}{y}$ and $y=2$ when $x=1$, then, when $x=2$, $y=$

(A) $\sqrt{\frac{27}{2}}$ (B) $\sqrt{\frac{27}{8}}$ (C) $\pm\sqrt{\frac{27}{8}}$ (D) $\pm\frac{3}{2}$ (E) $\pm\sqrt{\frac{27}{2}}$

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