Step 4: Now we can integrate:

$$\int\left(u^{\frac{3}{2}}-3u^{\frac{1}{2}}\right)du=\frac{2}{5}u^{\frac{5}{2}}-3\cdot\frac{2}{3}u^{\frac{3}{2}}+C$$

Step 5: Substituting back, we get:

$$\frac{2}{5}u^{\frac{5}{2}}-3\cdot\frac{2}{3}u^{\frac{3}{2}}+C=\frac{2}{5}(x+3)^{\frac{5}{2}}-2(x+3)^{\frac{3}{2}}+C$$

The answer is (C).

Problem 28. If $f(x)=\ln(\ln(1-x))$, then $f'(x)=$

Here, we use the chain rule.

Step 1: First, take the derivative of the outside function.

The derivative of $\ln u$ is $\frac{du}{u}$.

We get:

$$\frac{d}{dx}\ln(\ln(\ ))=\frac{1}{\ln(\ )}$$

Step 2: Now we take the derivative of the function in the denominator. Once again, the function is $\ln u$.

We get:

$$\frac{d}{dx}\ln(\ln(1-x))=\frac{1}{\ln(1-x)}\cdot\frac{-1}{1-x}=-\frac{1}{(1-x)\ln(1-x)}.$$

The answer is (D).

Problem 29. $\int_0^{\frac{\pi}{4}}\sin x\ dx+\int_{\frac{-\pi}{4}}^{0}\cos x\ dx=$

This is a pair of basic Trigonometric Integrals. You should have memorized several trigonometric integrals, particularly $\int\sin x\ dx=-\cos x+C$ and $\int\cos x\ dx=\sin x+C$

Step 1: $\int_0^{\frac{\pi}{4}}\sin x\ dx+\int_{\frac{-\pi}{4}}^{0}\cos x\ dx=-\cos x\Big|_0^{\frac{\pi}{4}}+\sin x\Big|_{\frac{-\pi}{4}}^{0}$

Step 2: Now we evaluate the limits of integration, and we're done.

$$-\cos x\Big|_0^{\frac{\pi}{4}}+\sin x\Big|_{-\frac{\pi}{4}}^{0}=\left[\left(-\cos\frac{\pi}{4}\right)-\left(-\cos(0)\right)\right]+\left[\left(\sin(0)\right)-\left(\sin\left(-\frac{\pi}{4}\right)\right)\right]=-\frac{1}{\sqrt{2}}+1+0+\frac{1}{\sqrt{2}}=1$$

The answer is (D).

PROBLEM 30. Boats A and B leave the same place at the same time. Boat A heads due north at 12 km/hr. Boat B heads due east at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)?

Step 1: The boats are moving at right angles to each other and are thus forming a right triangle with the distance between them forming the hypotenuse. Whenever we see right triangles in related rates problems, we look to use the Pythagorean Theorem. Call the distance that Boat A travels y, and the distance that Boat B travels x. Then the rate at which Boat A goes north is $\frac{dy}{dt}$, and the rate at which Boat B travels is $\frac{dx}{dt}$. The distance between the two boats is z, and we are looking for how fast z is growing, which is $\frac{dz}{dt}$. Now we can use the Pythagorean Theorem to set up the relationship: $x^2 + y^2 = z^2$

Step 2: Differentiating both sides we obtain:

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt} \text{ or } x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}.$$

Step 3: After 2.5 hours, Boat A has traveled 30 km and Boat B has traveled 45 km. Because of the Pythagorean Theorem, we also know that, when $y = 30$ and $x = 45$, $z = 54.08$.

Step 4: Now we plug everything into the equation from Step 2 and solve for $\frac{dz}{dt}$:

$$(45)(18)+(30)(12)=(54.08)\frac{dz}{dt}$$

$$1170=(54.08)\frac{dz}{dt}$$

$$21.63=\frac{dz}{dt}$$

The answer is (A).

PROBLEM 31. $\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{6}+h\right)-\tan\left(\frac{\pi}{6}\right)}{h}=$

This may *appear* to be a limit problem, but it is *actually* testing to see whether you know The Definition of the Derivative.

Step 1: You should recall that the Definition of the Derivative says

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x).$$

Thus, if we replace $f(x)$ with tan (x), we can rewrite the problem as:

$$\lim_{h\to 0}\frac{\tan(x+h)-\tan(x)}{h}=\left[\tan(x)\right]'.$$

Step 2: The derivative of tan x is $\sec^2 x$. Thus

$$\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{6}+h\right)-\tan\left(\frac{\pi}{6}\right)}{h}=\sec^2\left(\frac{\pi}{6}\right).$$

Step 3: Because $\sec\left(\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}},\sec^2\left(\frac{\pi}{6}\right)=\frac{4}{3}.$

The answer is (B).

> **Note:** If you had trouble with this problem, you should review the units on The Definition of the Derivative and Derivatives of Trigonometric Functions.

PROBLEM 32. If $\int_{30}^{100}f(x)dx=A$ and $\int_{50}^{100}f(x)dx=B$ then $\int_{30}^{50}f(x)dx=$

This question is testing your knowledge of the rules of Definite Integrals.

Step 1: Generally speaking, $\int_a^b f(x)dx+\int_b^c f(x)dx=\int_a^c f(x)dx.$

So here, $\int_{30}^{50}f(x)dx+\int_{50}^{100}f(x)dx=\int_{30}^{100}f(x)dx.$

If we substitute $\int_{30}^{100}f(x)dx=A$ and $\int_{50}^{100}f(x)dx=B,$ we get $\int_{30}^{50}f(x)dx+B=A.$

The answer is (B).

Problem 33. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

This problem requires you to know how to find the Derivative of an Inverse Function.

Step 1: The rule for finding the derivative of an inverse function is:

$$\text{If } y = f(x) \text{ and if } g(x) = f^{-1}(x) \text{ then } g'(x) = \frac{1}{f'(y)}.$$

Step 2: In order to use the formula, we need to find the derivative of f and the value of x that corresponds to $y = 10$.

First, $f'(x) = 6x - 1$. Second, when $y = 10$ we get $10 = 3x^2 - x$.

If we solve this for x we get $x = 2$ (and $x = -\frac{5}{3}$ but we'll use 2. It's easier.)

Step 3: Plugging into the formula, we get $\frac{1}{f'(y)} = \frac{1}{(6)(2)-1} = \frac{1}{11}$.

The answer is (E).

> **Note:** There was another possible answer using $x = -\frac{5}{3}$, but that doesn't give us one of the answer choices. Generally, the AP examination sticks to the easier answer. They are testing whether you know what to do and usually NOT trying to trick you.

Problem 34. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at

This is another Maxima/Minima question.

Step 1: Take the derivative of the function and set it equal to zero.

$$f'(x) = 3x^2 - 10x + 4 = 0$$

Step 2: Use the quadratic formula to solve for x. You should get $x = \{2.87, 0.46\}$.

Step 3: Now take the second derivative of the function.

$$f''(x) = 6x - 10$$

Step 4: Plug each of the critical values from Step 2 into the second derivative. If you get a positive value, the point is a minimum. If you get a negative value, the point is a maximum. If you get zero, the point is probably a point of inflection (don't worry about that here).

$$f''(2.87) = 7.21$$
$$f''(.46) = -7.21$$

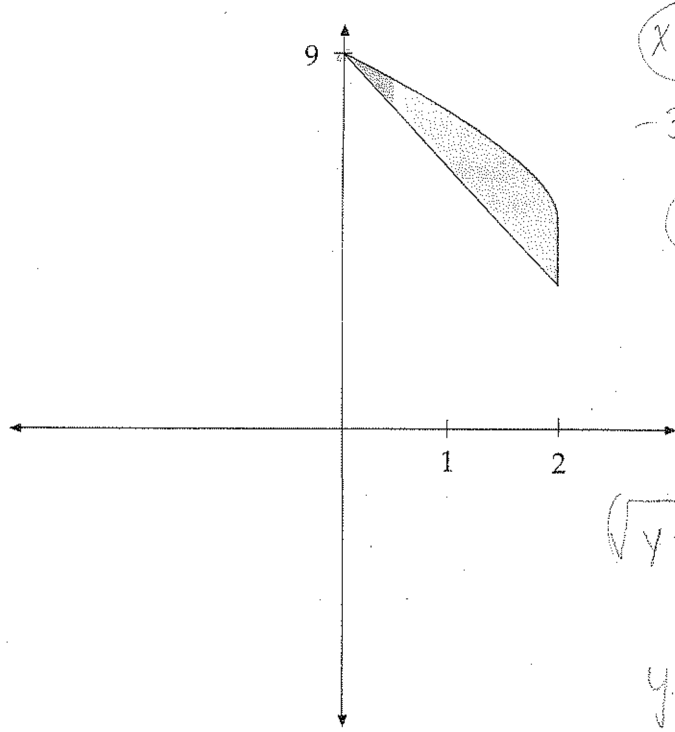
So 2.87 is the x-coordinate of the minimum. To find the y-coordinate, just plug 2.87 into $f(x)$ and you get – 4.06.

The answer is (C).

Problem 35. The volume generated by revolving about the y-axis the region enclosed by the graphs $y = 9 - x^2$ and $y = 9 - 3x$, for $0 \le x \le 2$, is

This is another Volume of a Solid of Revolution problem. As you should have noticed by now, these are very popular on the AP Examination and show up in both the multiple-choice section and in the Long Problem section. If you are not good at these, go back and review the unit carefully. You cannot afford to get these wrong on the AP! The good thing about *this* volume problem is that it is in the calculator part of the multiple choice section, so you can use a graphing calculator to assist you.

Step 1: First, graph the two curves on the same set of axes. The graph should look like this:



$x = \sqrt{y-9}$

$-3x = y - 9$

$x = -\frac{1}{3}y + 3$

$(\sqrt{y-9})^2 = (-\frac{1}{3}y + 3)^2$

$y - 9 = (-\frac{1}{3}y + 3)(-\frac{1}{3}y + 3)$

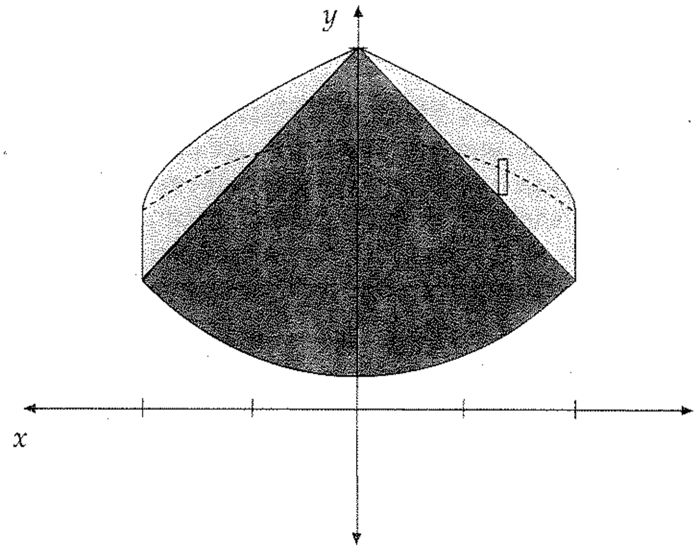
$y - 9 = \frac{1}{9}y^2 - y - y + 9$

$y - 9 = \frac{1}{9}y^2 - 2y + 9$

$0 = \frac{1}{9}y^2 - 3y + 18$

$\pi \int_3^9 (\frac{1}{9}y^2 - 3y + 18)\,dy$

Step 2: We are being asked to rotate this region around the y-axis, and both of the functions are in terms of x, so we should use the method of shells. We use this method whenever we take a vertical slice of a region and rotate it around an axis parallel to the slice (review the unit if you are not sure what it means). This will give us a region that looks like this:



Step 3: The formula for the method of shells says that if you have a region between two curves, $f(x)$ and $g(x)$ from $x = a$ to $x = b$, then the volume generated when the region is revolved around the y-axis is: $2\pi\int_a^b x[f(x)-g(x)]dx$; if $f(x)$ is above $g(x)$ throughout the region.Thus our integral is.

$$2\pi\int_0^2 x[(9-x^2)-(9-3x)]dx$$

We can simplify this integral to $2\pi\int_0^2 x(3x-x^2)dx = 2\pi\int_0^2(3x^2-x^3)dx$.

Step 4: Evaluate the integral:

$$2\pi\int_0^2(3x^2-x^3)dx = 2\pi\left(x^3-\frac{x^4}{4}\right)\Bigg|_0^2 = 8\pi$$

The answer is (C).

Problem 36. The average value of the function $f(x) = \ln^2 x$ on the interval [2,4] is

This problem requires you to be familiar with the Mean Value Theorem for Integrals which we use to find the average value of a function.

Step 1: If you want to find the average value of $f(x)$ on an interval $[a, b]$, you need to evaluate the integral. $\frac{1}{b-a}\int_a^b f(x)dx$ So here we evaluate the integral $\frac{1}{2}\int_2^4 \ln^2 x\ dx$.

You have to do this integral on your calculator because you do not know how to evaluate this integral analytically unless you are very good with integration by parts!

Use **fnint**. Divide this by 2 and you will get 1.204.

The answer is (B).

PROBLEM 37. $\frac{d}{dx}\int_0^{3x}\cos(t)dt =$

This problem is testing your knowledge of the Second Fundamental Theorem of Calculus. The theorem states that $\frac{d}{dx}\int_a^{u} f(t)dt = f(u)\frac{du}{dx}$, where a is a constant and u is a function of x. So all we have to do is follow the theorem. $\frac{d}{dx}\int_0^{3x}\cos(t)dt = 3\cos 3x$.

The answer is (E).

PROBLEM 38. If the definite integral $\int_1^3 (x^2+1)dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

This problem will require you to be familiar with the Trapezoid Rule. This is very easy to do on the calculator, and some of you may even have written programs to evaluate this. Even if you haven't, the formula is easy. The area under a curve from $x = a$ to $x = b$, divided into n intervals is approximated by the Trapezoid Rule and is

$$\left(\frac{1}{2}\right)\left(\frac{b-a}{n}\right)\left[y_0 + 2y_1 + 2y_2 + 2y_3 \ldots + 2y_{n-2} + 2y_{n-1} + y_n\right]$$

This formula may look scary, but it actually is quite simple, and the AP Examination never uses a very large value for n anyway.

Step 1: $\frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$. Plugging into the formula, we get:

$$\frac{1}{4}\left[\left(1^2+1\right) + 2\left(1.5^2+1\right) + 2\left(2^2+1\right) + 2\left(2.5^2+1\right) + \left(3^2+1\right)\right].$$

This is easy to Plug Into your calculator and you will get 10.75 or $\frac{43}{4}$.

Step 2: In order to find the error, we now need to know the actual value of the integral.

$$\int_1^3 x^2 + 1dx = \frac{x^3}{3} + x\Big|_1^3 = \frac{32}{3} \text{ or } 10.666.$$

Step 3: The error is $\frac{43}{4} - \frac{32}{3} = \frac{1}{12}$.

The answer is (C).

PROBLEM 39. The radius of a sphere is increasing at a rate proportional to itself. If the radius is 4 initially, and the radius is 10 after two seconds, then what will the radius be after three seconds?

This is not a Related Rate problem, this is a Differential Equation! It just happens to involve a rate.

Step 1: If we translate the first sentence into an equation we get: $\frac{dR}{dt} = kR$.

Put all of the terms that contain an R on the left of the equals sign, and all of the terms that contain a t on the right hand side. $\frac{dR}{R} = kdt$

Step 2: Integrate both sides: $\int \frac{dR}{R} = k\int dt$.

Step 3: If we solve this for R we get $R = Ce^{kt}$ (see the Unit on Differential Equations).

Now we need to solve for C and k. First we solve for C by Plugging In the information that the radius is 4 initially. This means that $R = 4$ when $t = 0$.

$$4 = Ce^0 \text{ then } C = 4.$$

Next we solve for k by Plugging In the information that $R = 10$ when $t = 2$.

$$\begin{aligned}
10 &= 4e^{2k} \\
\frac{5}{2} &= e^{2k} \\
\ln\frac{5}{2} &= 2k \\
\frac{1}{2}\ln\frac{5}{2} &= k
\end{aligned}$$

Step 4: Now we have our final equation: $R = 4e^{\left(\frac{1}{2}\ln\frac{5}{2}\right)t}$.

If we Plug In $t = 3$ we get: $R = 4e^{\left(\frac{1}{2}\ln\frac{5}{2}\right)(3)} = 15.811$

The answer is (C).

Problem 40. Use differentials to approximate the change in the volume of a sphere when the radius is increased from 10 to 10.02 cm.

The volume of a sphere is $V = \frac{4}{3}\pi R^3$. Using differentials, the change will be: $dV = 4\pi R^2 dR$.

Substitute in $R = 10$ and $dR = .02$, and we get:

$$dV = 4\pi\left(10^2\right)(.02)$$

$$dV = 8\pi \approx 25.133 cm^3$$

The answer is (E).

Problem 41. $\int \ln 2x \;\; dx =$

This is a simple integral that we do using Integration By Parts. The AB Examination only has the simplest of these types of integrals, although the BC Examination has harder ones. Furthermore, you should memorize that $\int \ln(ax) \;\; dx = x\ln(ax) - x + C$, which makes this integral easy.

Step 1: The formula for Integration By Parts is: $\int udv = uv - \int vdu$

The trick is that we have to let $dv = dx$.

$$\text{Let } u = \ln 2x \qquad \text{and } dv = dx$$

$$du = \frac{2}{2x}dx = \frac{1}{x}dx \text{ and } v = x$$

Plugging in to the formula we get:

$$\int \ln 2x dx = x\ln 2x - \int dx = x\ln 2x - x + C.$$

The answer is (D).

Problem 42. For the function $f(x) = \begin{cases} ax^3 - 6x; \text{if } \; x \leq 1 \\ bx^2 + 4; x > 1 \end{cases}$ to be continuous and differentiable, a must be

This question is testing your knowledge of the rules of Continuity, where we also discuss differentiability.

Step 1: If the function is continuous, then if we plug 1 into the top and bottom pieces of the function we should get the same answer.

$$a(1^3) - 6(1) = b(1^2) + 4$$
$$a - 6 = b + 4$$

Step 2: If the function is differentiable, then if we plug 1 into the derivatives of the top and bottom pieces of the function we should get the same answer.

$$3a(1^2) - 6 = 2b(1)$$
$$3a - 6 = 2b$$

Step 3: Now we have a pair of simultaneous equations. If we solve them, we get $a = -14$

The answer is (C).

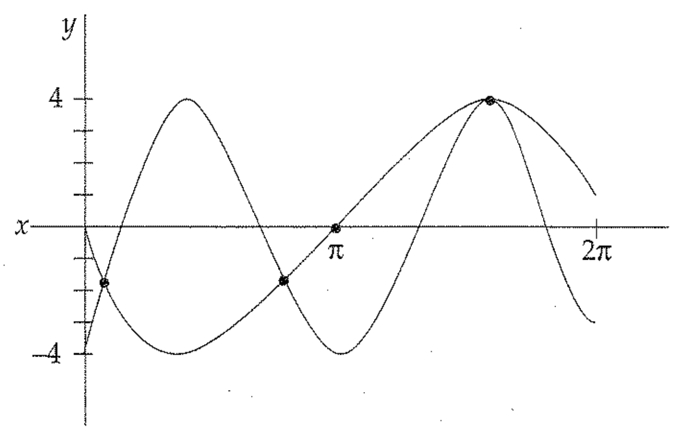
Problem 43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?

If you want to find acceleration, all you have to do is take the second derivative of the position functions.

Step 1: $\frac{dy_1}{dx} = -2\sin 2t$ and $\frac{dy_2}{dx} = 4\cos t$

$\frac{d^2y_1}{dx^2} = -4\cos 2t$ and $\frac{d^2y_2}{dx^2} = -4\sin t$

Step 2: Now all we have to do is to graph both of these equations on the same set of axes on a calculator. You should make the window from $x = 0$ to $x = 7$ (leave yourself a little room so that you can see the whole range that you need). You should get a picture that looks like this:



Where the graphs intersect, the acceleration is the same. There are three points of intersection.

The answer is (D).

PROBLEM 44. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$; where t stands for time.

Step 1: If we want to find the distance traveled, we take the integral of velocity from the starting time to the finishing time. Therefore, we need to evaluate $\int_0^4 7e^{-t^2}\,dt$.

Step 2: But we have a problem! We can't take the integral of e^{-t^2}. This means that the AP wants you to find the answer using your calculator.

Rounded to three decimal places, the answer is 6.204.

The answer is (B).

PROBLEM 45. $\int \tan^6 x \sec^2 x\,dx =$

We can do this integral with u-substitution.

Step 1: Let $u = \tan x$. Then $du = \sec^2 x\,dx$.

Step 2: Substituting, we get: $\int \tan^6 x \sec^2 x\,dx = \int u^6 du$.

Step 3: This is an easy integral: $\int u^6 du = \frac{u^7}{7} + C$.

Step 4: Substituting back, we get: $\frac{\tan^7 x}{7} + C$.

The answer is (A).