CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—40

Percent of total grade—50

Part A consists of 28 questions that will be answered on side 1 of the answer sheet. Following are the directions for Section I, Part A.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = 5x^{\frac{4}{3}}$, then $f'(8) =$

(A) 10 (B) $\frac{40}{3}$ (C) 40 (D) 80 (E) $\frac{160}{3}$

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2. $\lim_{x\to\infty} \frac{5x^2-3x+1}{4x^2+2x+5}$ is

(A) 0 (B) $\frac{4}{5}$ (C) $\frac{3}{11}$ (D) $\frac{5}{4}$ (E) ∞

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3. If $f(x) = \dfrac{3x^2 + x}{3x^2 - x}$ then $f'(x)$ is

(A) 1

(B) $\dfrac{6x^2 + 1}{6x^2 - 1}$

(C) $\dfrac{-6}{(3x - 1)^2}$

(D) $\dfrac{-2x^2}{(x^2 - x)^2}$

(E) $\dfrac{36x^3 - 2x}{(x^2 - x)^2}$

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4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

(A) 1 (B) $\dfrac{8}{7}$ (C) -1 (D) 0 (E) undefined

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5. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$

(A) $\frac{8 + 2y - 2x}{6y - 2x}$

(B) $\frac{3y - x}{y - x}$

(C) $\frac{2x - 2y}{6y - 2x}$

(D) $\frac{1}{3}$

(E) $\frac{y - x}{3y - x}$

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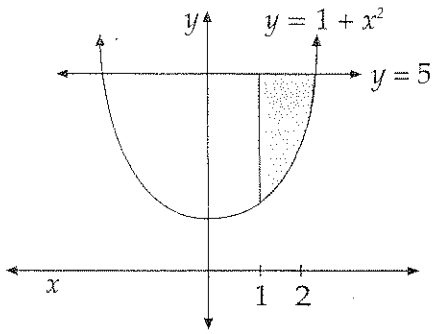

6. Which of the following integrals correctly corresponds to the area of the shaded region in the figure above ?

(A) $\int_1^2 (x^2 - 4)dx$

(B) $\int_1^2 (4 - x^2)dx$

(C) $\int_1^5 (x^2 - 4)dx$

(D) $\int_1^5 (x^2 + 4)dx$

(E) $\int_1^5 (4 - x^2)dx$

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7. If $f(x) = \sec x + \csc x$, then $f'(x) =$

(A) 0

(B) $\sec^2 x + \csc^2 x$

(C) $\csc x - \sec x$

(D) $\sec x \tan x + \csc x \cot x$

(E) $\sec x \tan x - \csc x \cot x$

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8. An equation of the line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at (2, 4) is

(A) $-4x + y = 20$ (B) $,4x + 7y = 20$ (C) $-7x + 4y = 2$ (D) $7x + 4y = 30$ (E) $4x + 7y = 36$

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9. $\int_{-1}^{1} \frac{4}{1+x^2} dx =$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

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10. If $f(x) = \cos^2 x$, then $f''(\pi) =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 2π

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11. If $f(x) = \frac{5}{x^2+1}$ and $g(x) = 3x$, then $g(f(2)) =$

(A) -3 (B) $\frac{5}{37}$ (C) 3 (D) 5 (E) $\frac{37}{5}$

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12. $\int x\sqrt{5x^2-4}\ dx =$

(A) $\frac{1}{10}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(B) $\frac{1}{15}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(C) $-\frac{1}{5}\left(5x^2-4\right)^{-\frac{1}{2}}+C$

(D) $\frac{20}{3}\left(5x^2-4\right)^{\frac{3}{2}}+C$

(E) $\frac{3}{20}\left(5x^2-4\right)^{\frac{3}{2}}+C$

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13. The slope of the line tangent to the graph of $3x^2 + 5 \ln y = 12$ at (2, 1) is

(A) $-\frac{12}{5}$ (B) $\frac{12}{5}$ (C) $\frac{5}{12}$ (D) 12 (E) –7

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14. The equation $y = 2 - 3\sin\frac{\pi}{4}(x-1)$ has a fundamental period of

(A) $\frac{1}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{4}{\pi}$ (D) 8 (E) 2π

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15. If $f(x) = \begin{cases} x^2 + 5 \text{ if } x < 2 \\ 7x - 5 \text{ if } x \geq 2 \end{cases}$, for all real numbers x, which of the following must be true?

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x = 2$.

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, and III

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16. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

(A) 10 (B) 4 (C) 3 (D) –4 (E) –10

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17. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at

(A) $t = 1$
(B) $t = 3$
(C) $t = 5$
(D) $t = 1$ and $t = 5$
(E) $t = 1$ and $t = 3$ and $t = 5$

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18. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

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19. $\int \left(e^{3\ln x} + e^{3x}\right)\, dx =$

(A) $3 + \frac{e^{3x}}{3} + C$

(B) $\frac{x^4}{4} + 3e^{3x} + C$

(C) $\frac{e^{x^4}}{4} + 3e^{3x} + C$

(D) $\frac{e^{x^4}}{4} + \frac{e^{3x}}{3} + C$

(E) $\frac{x^4}{4} + \frac{e^{3x}}{3} + C$

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20. If $f(x) = \sqrt{(x^3 + 5x + 121)}\,(x^2 + x + 11)$ then $f'(0) =$

(A) $\frac{5}{2}$ (B) $\frac{27}{2}$ (C) 22 (D) $22 + \frac{2}{\sqrt{5}}$ (E) $\frac{247}{2}$

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21. If $f(x) = 5^{3x}$ then $f'(x) =$

(A) $5^{3x}(\ln 125)$ (B) $\frac{5^{3x}}{3 \ln 5}$ (C) $3(5^{2x})$ (D) $3(5^{3x})$ (E) $3x(5^{3x-1})$

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22. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi\int_1^8 \left(x^2+1\right)^3 dx$

(B) $\pi\int_1^8 \left(x^2+1\right)^6 dx$

(C) $\pi\int_0^1 \left(x^2+1\right)^3 dx$

(D) $\pi\int_0^1 \left(x^2+1\right)^6 dx$

(E) $2\pi\int_0^1 \left(x^2+1\right)^6 dx$

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23. $\lim_{x\to 0} 4\,\dfrac{\sin x\cos x - \sin x}{x^2} =$

(A) 2 (B) $\dfrac{40}{3}$ (C) ∞ (D) 0 (E) undefined

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24. If $\dfrac{dy}{dx} = \dfrac{(3x^2+2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18 (B) $\sqrt{66}$ (C) 58 (D) $\sqrt{74}$ (E) $\sqrt{58}$

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25. $\displaystyle\int \frac{dx}{9+x^2} =$

(A) $3\tan^{-1}\left(\dfrac{x}{3}\right)+C$

(B) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{x}{3}\right)+C$

(C) $\dfrac{1}{9}\tan^{-1}\left(\dfrac{x}{3}\right)+C$

(D) $\dfrac{1}{3}\tan^{-1}(x)+C$

(E) $\dfrac{1}{9}\tan^{-1}(x)+C$

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26. If $f(x)=\cos^3(x+1)$ then $f'(\pi)=$

(A) $-3\cos^2(\pi+1)\sin(\pi+1)$

(B) $3\cos^2(\pi+1)$

(C) $3\cos^2(\pi+1)\sin(\pi+1)$

(D) $3\pi\cos^2(\pi+1)$

(E) 0

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27. $\int x\sqrt{x+3}\,dx=$

(A) $\frac{2}{3}(x)^{\frac{3}{2}}+6(x)^{\frac{1}{2}}+C$

(B) $\frac{2(x+3)^{\frac{3}{2}}}{3}+C$

(C) $\frac{2}{5}(x+3)^{\frac{5}{2}}-2(x+3)^{\frac{3}{2}}+C$

(D) $\frac{3(x+3)^{\frac{3}{2}}}{2}+C$

(E) $\frac{4x^2(x+3)^{\frac{3}{2}}}{3}+C$

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28. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$

(A) $-\dfrac{1}{\ln(1-x)}$

(B) $\dfrac{1}{(1-x)\ln(1-x)}$

(C) $\dfrac{1}{(1-x)^2}$

(D) $-\dfrac{1}{(1-x)\ln(1-x)}$

(E) $-\dfrac{1}{\ln(1-x)^2}$

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