

ANSWERS AND EXPLANATIONS TO SECTION I

PROBLEM 1. If $f(x) = 5x^{\frac{4}{3}}$, then $f'(8) =$

We need to use Basic Differentiation to solve this problem.

Step 1: $f'(x) = \frac{4}{3} \left(5x^{\frac{1}{3}} \right)$

Step 2: Now all we have to do is Plug In 8 for x and simplify.

$$\frac{4}{3} \left(5 \left(8^{\frac{1}{3}} \right) \right) = \frac{4}{3} (5(2)) = \frac{40}{3}$$

The answer is (B).

PROBLEM 2. $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5}$ is

Step 1: To solve this problem, you need to remember how to evaluate Limits. Always do limit problems on the first pass. Whenever we have a limit of a polynomial fraction where $x \rightarrow \infty$, we divide the numerator and the denominator, separately, by the highest power of x in the fraction.

$$\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5} = \lim_{x \rightarrow \infty} \frac{\frac{5x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{4x^2}{x^2} + \frac{2x}{x^2} + \frac{5}{x^2}}$$

Step 2: Simplify $\lim_{x \rightarrow \infty} \frac{5 - \frac{3}{x} + \frac{1}{x^2}}{4 + \frac{2}{x} + \frac{5}{x^2}}$

Step 3: Now take the limit. Remember that the $\lim_{x \rightarrow \infty} \frac{k}{x^n} = 0$, if $n > 0$, where k is a constant. Thus we get

$$\lim_{x \rightarrow \infty} \frac{5 - \frac{3}{x} + \frac{1}{x^2}}{4 + \frac{2}{x} + \frac{5}{x^2}} = \lim_{x \rightarrow \infty} \frac{5 - 0 + 0}{4 + 0 + 0} = \frac{5}{4}$$

The answer is (D).

PROBLEM 3. If $f(x) = \frac{3x^2 + x}{3x^2 - x}$ then $f'(x)$ is

Step 1: We need to use the Quotient Rule to evaluate this derivative. Remember,

the derivative of $\frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$. But before we take the derivative, we should factor an x out of the top and bottom and cancel, simplifying the quotient.

$$f(x) = \frac{3x^2 + x}{3x^2 - x} = \frac{x(3x+1)}{x(3x-1)} = \frac{3x+1}{3x-1}$$

Step 2: Now take the derivative.

$$f'(x) = \frac{(3x-1)(3) - (3x+1)(3)}{(3x-1)^2}$$

Step 3: Simplify:

$$\frac{9x-3-9x-3}{(3x-1)^2} = \frac{-6}{(3x-1)^2}$$

The answer is (C).

PROBLEM 4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

This problem is testing your knowledge of Continuity.

Step 1: Notice that if we plug 4 into the numerator and denominator we get $\frac{0}{0}$, which is undefined. So, the first thing that we should do is factor the numerator. What we are looking for is a common factor in the numerator and denominator. If we find a common factor, we can cancel the factors and simplify the problem.

$$\text{We get } f(x) = \frac{x^2 - 7x + 12}{x - 4} = \frac{(x-3)(x-4)}{x-4} = (x-3).$$

Step 2: Now we Plug In 4 for x and we get 1.

The answer is (A).

PROBLEM 5. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$

Whenever we have a polynomial where the x 's and y 's are not separated we need to use Implicit Differentiation to find the derivative.

Step 1: Take the derivative of everything with respect to x .

$$2x \frac{dx}{dx} - 2 \left(x \frac{dy}{dx} + y \frac{dx}{dx} \right) + 6y \frac{dy}{dx} = 0 \quad \text{Remember that } \frac{dx}{dx} = 1!$$

Step 2: Simplify and then put all of the terms containing $\frac{dy}{dx}$ on one side, and all of the other terms on the other side.

$$2x - 2x \frac{dy}{dx} - 2y + 6y \frac{dy}{dx} = 0$$

$$-2x \frac{dy}{dx} + 6y \frac{dy}{dx} = 2y - 2x$$

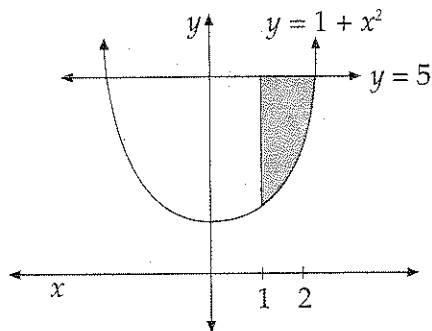
Factor out the $\frac{dy}{dx}$, then isolate it.

$$\frac{dy}{dx} (6y - 2x) = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y - 2x}{(6y - 2x)} = \frac{y - x}{3y - x}$$

The answer is (E).

PROBLEM 6.



Which of the following integrals correctly corresponds to the area of the region in the figure above between the curve $y = 1 + x^2$ and the line $y = 5$ from $x = 1$ to $x = 2$?

We use integrals to find the Area Between Two Curves. If the top curve of a region is $f(x)$ and the bottom curve of a region is $g(x)$, from $x = a$ to $x = b$, then the area is found by the integral

$$\int_a^b [f(x) - g(x)] dx$$

Step 1: The top curve here is the line $y = 5$, and the bottom curve is $y = 1 + x^2$, and the region extends from the line $x = 1$ to the line $x = 2$. Thus, the integral for the area is

$$\int_1^2 [(5) - (1 + x^2)] dx = \int_1^2 (4 - x^2) dx$$

The answer is (B).

PROBLEM 7. If $f(x) = \sec x + \csc x$, then $f'(x) =$

This question is testing whether you know your Derivatives of Trigonometric Functions. If you do, this is an easy problem.

Step 1: The derivative of $\sec x$ is $\sec x \tan x$ and the derivative of $\csc x$ is $-\csc x \cot x$. That makes the derivative here $\sec x \tan x - \csc x \cot x$.

The answer is (E).

PROBLEM 8. An equation of the line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at $(2, 4)$ is

Here we do everything that we normally do for finding the Equations of Tangent Lines, except that we use the negative reciprocal of the slope to find the normal line. This is because the normal line is perpendicular to the tangent line.

Step 1: First, find the slope of the tangent line.

$$\frac{dy}{dx} = \frac{1}{2}(3x^2 + 2x)^{-\frac{1}{2}}(6x + 2)$$

Step 2: DON'T SIMPLIFY. Immediately Plug In $x = 2$. We get:

$$\frac{dy}{dx} = \frac{1}{2}(3x^2 + 2x)^{-\frac{1}{2}}(6x + 2) = \frac{1}{2}(3(2)^2 + 2(2))^{-\frac{1}{2}}(6(2) + 2) = \frac{1}{2}(16)^{-\frac{1}{2}}(14) = \frac{7}{4}$$

This means that the slope of the tangent line at $x = 2$ is $\frac{7}{4}$, so the slope of the normal line is $-\frac{4}{7}$.

Step 3: Then the equation of the tangent line is $(y - 4) = -\frac{4}{7}(x - 2)$.

Step 4: Multiply through by 7 and simplify.

$$7y - 28 = -4x + 8$$

$$4x + 7y = 36$$

The answer is (E).

PROBLEM 9. $\int_{-1}^1 \frac{4}{1+x^2} dx =$

You should recognize this integral as one of the Inverse Trigonometric Integrals.

Step 1: As you should recall, $\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$. The 4 is no big deal, just multiply the integral by 4 to get $4 \tan^{-1}(x)$. Then we just have to evaluate the limits of integration.

Step 2: $4 \tan^{-1}(x) \Big|_{-1}^1 = 4 \tan^{-1}(1) - 4 \tan^{-1}(-1) = 4\left(\frac{\pi}{4}\right) - 4\left(-\frac{\pi}{4}\right) = 2\pi$

The answer is (D).

PROBLEM 10. If $f(x) = \cos^2 x$, then $f''(\pi) =$

This problem is just asking us to find a higher order Derivative of a Trigonometric Function.

Step 1: The first derivative requires the chain rule:

$$f(x) = \cos^2 x$$

$$f'(x) = 2(\cos x)(-\sin x) = -2 \cos x \sin x$$

Step 2: The second derivative requires the product rule:

$$f'(x) = -2 \cos x \sin x$$

$$f''(x) = -2(\cos x \cos x - \sin x \sin x) = -2(\cos^2 x - \sin^2 x)$$

Step 3: Now we Plug In π for x and simplify.

$$-2(\cos^2(\pi) - \sin^2(\pi)) = -2(1 - 0) = -2$$

The answer is (A).

PROBLEM 11. If $f(x) = \frac{5}{x^2+1}$ and $g(x) = 3x$ then $g(f(2)) =$

Step 1: To find $g(f(x))$, all you need to do is to replace all of the x 's in $g(x)$ with $f(x)$'s.

$$g(f(x)) = 3f(x) = 3\left(\frac{5}{x^2+1}\right) = \frac{15}{x^2+1}$$

Step 2: Now all we have to do is Plug In 2 for x .

$$g(f(2)) = \frac{15}{2^2+1} = 3$$

The answer is (C).

PROBLEM 12. $\int x\sqrt{5x^2-4} dx =$

Any time we have an integral with an x factor whose power is one less than another x factor, we can try to do the integral with u -substitution. This is our favorite technique for doing integration and the most important one to master.

Step 1: Let $u = 5x^2 - 4$ and $du = 10xdx$ and so $\frac{1}{10} du = xdx$.

Then we can rewrite the integral as:

$$\int x\sqrt{5x^2-4} dx = \frac{1}{10} \int u^{\frac{1}{2}} du.$$

Step 2: Now this becomes a basic integral.

$$\frac{1}{10} \int u^{\frac{1}{2}} du = \frac{1}{10} \left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) + C = \frac{1}{15} u^{\frac{3}{2}} + C$$

Step 3: Reverse the substitution and we get: $\frac{1}{15} (5x^2 - 4)^{\frac{3}{2}} + C$

The answer is (B).

PROBLEM 13. The slope of the line tangent to the graph of $3x^2 + 5\ln y = 12$ at $(2,1)$ is

This is another Equation of a Tangent Line problem, combined with Implicit Differentiation. Often the AP Examination has more than one tangent line problem, so make sure that you can do these well!

By the way, do you remember the derivative of $\ln(f(x))$? It is $\frac{f'(x)}{f(x)}$.

Step 1: First, we take the derivative of the equation.

$$6x \frac{dx}{dx} + \frac{5}{y} \frac{dy}{dx} = 0$$

Step 2: Next, we simplify and solve for $\frac{dy}{dx}$.

$$6x + \frac{5}{y} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6xy}{5}$$

Step 3: Now we Plug In 2 for x and 1 for y to get the slope of the tangent line.

$$\frac{dy}{dx} = \frac{-6(2)(1)}{5} = \frac{-12}{5}$$

The answer is (A).

Note: We could have plugged in directly for x and y after simplifying. On a more complicated derivative you ALWAYS want to Plug In right after you differentiate. On a simple one such as this, the choice is up to you.

PROBLEM 14. The equation $y = 2 - 3\sin \frac{\pi}{4}(x - 1)$ has a fundamental period of

The AP people expect you to remember a lot of your trigonometry, so if you're rusty, review the unit in the Appendix.

Step 1: In an equation of the form $f(x) = A \sin B(x \pm C) \pm D$, you should know four components. The amplitude of the equation is A , the horizontal or phase shift is $\pm C$, the vertical shift is $\pm D$, and the fundamental period is $\frac{2\pi}{B}$.

The same is true for $f(x) = A \cos B(x \pm C) \pm D$.

Step 2: All we have to do is Plug Into the formula for the period.

$$\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{4}} = 8$$

The answer is (D).

PROBLEM 15. If $f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 7x - 5 & \text{if } x \geq 2 \end{cases}$, for all real numbers x , which of the following must be true?

- I. $f(x)$ is continuous everywhere.
- II. $f(x)$ is differentiable everywhere.
- III. $f(x)$ has a local minimum at $x = 2$.

This problem is testing your knowledge of the rules of continuity and differentiability. While the more formal treatment is located in the unit on Continuity, here we'll go directly to a shortcut to the right answer. This type of function is called a piecewise function because it is broken into two or more pieces, depending on the value of x that one is looking at.

Step 1: If a piecewise function is continuous at a point a , then when you plug a into each of the pieces of the function, you should get the same answer. The function consists of a pair of polynomials (Remember that all polynomials are continuous!), where the only point that might be a problem is $x = 2$. So here we'll plug 2 into both pieces of the function to see if we get the same value. If we do, then the function is continuous. If we don't, then it's discontinuous. At $x = 2$, the upper piece is equal to 9 and the lower piece is also equal to 9. So the function is continuous everywhere, and I is true. You should then eliminate answer choice C.

Step 2: If a piecewise function is differentiable at a point a , then when you plug a into each of the derivatives of the pieces of the function, you should get the same answer. It is the same idea as in Step 1. So here we will plug 2 into the derivatives of both pieces of the function to see if we get the same value. If we do, then the function is differentiable. If we don't, then it is non-differentiable at $x = 2$.

The derivative of the upper piece is $2x$, and at $x = 2$, the derivative is 4.

The derivative of the lower piece is 7 everywhere.

Because the two derivatives are not equal, the function is not differentiable everywhere and II is false. You should then eliminate answer choices B and E.

Step 3: The slope of the function to the left of $x = 2$ is 4. The slope of the function to the right of $x = 2$ is 7. If the slope of a function has the same sign on either side of a point then the function cannot have a local minimum or maximum at that point. So III is false because of what we found in Step 2.

The answer is (A).

PROBLEM 16. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

This problem requires you to know how to find Maxima/Minima. This is a part of curve sketching and is one of the most important parts of Differential Calculus. A function has *critical points* where the derivative is zero or undefined (which is never a problem when the function is an ordinary polynomial). After finding the critical points we test them to determine whether they are maxima or minima or something else.

Step 1: First, as usual, take the derivative and set it equal to zero.

$$\begin{aligned}f'(x) &= 3x^2 - 18x - 120 \\3x^2 - 18x - 120 &= 0\end{aligned}$$

Step 2: Find the values of x that make the derivative equal to zero. These are the critical points.

$$\begin{aligned}3x^2 - 18x - 120 &= 0 \\x^2 - 6x - 40 &= 0 \\(x - 10)(x + 4) &= 0 \\x &= \{10, -4\}\end{aligned}$$

Step 3: In order to determine whether a critical point is a maximum or a minimum, we need to take the second derivative. $f''(x) = 6x - 18$

Step 4: Now we plug the critical points from Step 2 into the second derivative. If it yields a negative value, then the point is a maximum. If it yields a positive value, then the point is a minimum. If it yields zero, it is neither, and is most likely a point of inflection.

$$\begin{aligned}6(10) - 18 &= 42 \\6(-4) - 18 &= -42\end{aligned}$$

Therefore 10 is a minimum.

The answer is (A).

PROBLEM 17. The acceleration of a particle moving along the x -axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at

Step 1: Because acceleration is the derivative of velocity, if we know the acceleration of a particle, we can find the velocity by integrating the acceleration with respect to t .

$$\int (4t - 12) dt = 2t^2 - 12t + C$$

Next, because the velocity is 10 at $t = 0$, we can Plug In 0 for t and solve for the constant.

$$2(0)^2 - 12(0) + C = 10.$$

Therefore $C = 10$ and the velocity, $v(t)$, is $2t^2 - 12t + 10$

Step 2: In order to find when the particle is changing direction we need to know when the velocity is equal to zero, so we set $v(t) = 0$ and solve for t .

$$2t^2 - 12t + 10 = 0$$

$$t^2 - 6t + 5 = 0$$

$$(t - 5)(t - 1) = 0$$

$$t = \{1, 5\}$$

Now, provided that the acceleration is not also zero at $t = \{1, 5\}$, the particle will be changing direction at those times. The acceleration is found by differentiating the equation for velocity with respect to time: $a(t) = 4t - 12$. This is not zero at either $t = 1$ or $t = 5$. Therefore, the particle is changing direction when $t = 1$ and $t = 5$.

The answer is (D).

PROBLEM 18. The average value of the function $f(x) = (x - 1)^2$ on the interval from $x = 1$ to $x = 5$ is

Step 1: If you want to find the average value of $f(x)$ on an interval $[a, b]$ you need to evaluate the integral $\frac{1}{b-a} \int_a^b f(x) dx$.

So here we would evaluate the integral $\frac{1}{5-1} \int_1^5 (x-1)^2 dx$.

$$\begin{aligned} \text{Step 2: } \frac{1}{5-1} \int_1^5 (x-1)^2 dx &= \frac{1}{4} \int_1^5 (x^2 - 2x + 1) dx \\ &= \frac{1}{4} \left(\frac{x^3}{3} - x^2 + x \right) \Big|_1^5 = \frac{1}{4} \left[\left(\frac{5^3}{3} - 5^2 + 5 \right) - \left(\frac{1}{3} - 1 + 1 \right) \right] \\ &= \frac{1}{4} \left(\frac{125}{3} - 20 - \frac{1}{3} \right) = \frac{64}{12} = \frac{16}{3} \end{aligned}$$

The answer is (B).

PROBLEM 19. $\int (e^{3\ln x} + e^{3x}) dx =$

This problem requires that you know your rules of Exponential Functions.

Step 1: First of all, $e^{3\ln x} = e^{\ln x^3} = x^3$. So we can rewrite the integral as

$$\int (e^{3\ln x} + e^{3x}) dx = \int (x^3 + e^{3x}) dx.$$

Step 2: The rule for the integral of an exponential function is $\int e^k dx = \frac{1}{k} e^{kx} + C$.

Now we can do this integral. $\int (x^3 + e^{3x}) dx = \frac{x^4}{4} + \frac{1}{3} e^{3x} + C$.

The answer is (E).

PROBLEM 20. If $f(x) = \sqrt{(x^3 + 5x + 121)}(x^2 + x + 11)$ then $f'(0) =$

This problem is just a complicated derivative, requiring you to be familiar with the Chain Rule and the Product Rule.

Step 1: $f'(x) = \frac{1}{2}(x^3 + 5x + 121)^{-\frac{1}{2}}(3x^2 + 5)(x^2 + x + 11) + (x^3 + 5x + 121)^{\frac{1}{2}}(2x + 1)$

Step 2: Whenever a problem asks you to find the value of a complicated derivative at a particular point, NEVER simplify the derivative. Immediately Plug In the value for x and do arithmetic instead of algebra.

$$\begin{aligned} f'(x) &= \frac{1}{2}(0^3 + 5(0) + 121)^{-\frac{1}{2}}(3(0)^2 + 5)(0^2 + (0) + 11) + ((0)^3 + 5(0) + 121)^{\frac{1}{2}}(2(0) + 1) \\ &= \frac{1}{2}(121)^{-\frac{1}{2}}(5)(11) + (121)^{\frac{1}{2}}(1) = \frac{5}{2} + 11 = \frac{27}{2} \end{aligned}$$

The answer is (B).

PROBLEM 21. If $f(x) = 5^{3x}$ then $f'(x) =$

This problem requires you to know how to find the Derivative of an Exponential Function. The rule is: If a function is of the form $a^{f(x)}$, its derivative is $a^{f(x)}(\ln a) f'(x)$. Now all we have to do is follow the rule!

Step 1: $f(x) = 5^{3x}$
 $f'(x) = 5^{3x}(\ln 5)(3)$

Step 2: If you remember your rules of logarithms, $3\ln 5 = \ln(5^3) = \ln 125$.

So we can rewrite the answer to $f'(x) = 5^{3x}(\ln 5)(3) = 5^{3x} \ln 125$.

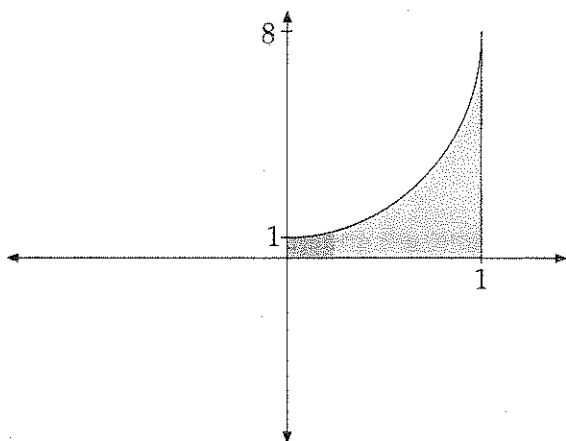
The answer is (A).

PROBLEM 22. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, and the x -axis, is revolved about the x -axis. Its volume is found by evaluating which of the following integrals?

This problem requires you to know how to find the Volume of a Solid of Revolution.

If you have a region between two curves, from $x = a$ to $x = b$, then the volume generated when the region is revolved around the x -axis is: $\pi \int_a^b [f(x)]^2 - [g(x)]^2 dx$, if $f(x)$ is above $g(x)$ throughout the region.

Step 1: First, we have to determine what the region looks like. The curve looks like this:



The shaded region is the part that we are interested in. Notice that the curve is always above the x -axis (which is $g(x)$). Now we just follow the formula:

$$\pi \int_0^1 \left[(x^2 + 1)^3 \right]^2 - [0]^2 dx = \pi \int_0^1 (x^2 + 1)^6 dx$$

The answer is (D).

PROBLEM 23. $\lim_{x \rightarrow 0} 4 \frac{\sin x \cos x - \sin x}{x^2} =$

This problem requires us to evaluate the Limit of a Trigonometric Function.

There are two important trigonometric limits to memorize:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

Step 1: The first step that we always take when evaluating the limit of a trigonometric function is to rearrange the function so that it looks like some combination of the limits above. We can do this by factoring a $\sin x$ out of the numerator.

Now we can break this into limits that we can easily evaluate.

$$\lim_{x \rightarrow 0} 4 \frac{\sin x \cos x - \sin x}{x^2} = 4 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{\cos x - 1}{x} \right)$$

$$\left(\text{Note that } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = -\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0. \right)$$

Step 2: Now if we take the limit as $x \rightarrow 0$ we get: $4(1)(0) = 0$

The answer is (D).

PROBLEM 24. If $\frac{dy}{dx} = \frac{(3x^2 + 2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

This is a very basic Differential Equation. As with many of the more difficult topics in Calculus, the AP examination only tends to ask us to solve very straightforward differential equations. In fact, on the AB examination, you are only going to need to know one technique for getting these right. It is called *Separation of Variables*.

Step 1: First, take all of the terms with a y in them and put them on the left side of the equal sign. Take all of the terms with an x in them and put them on the right side of the equal sign. Then we get:

$$y dy = (3x^2 + 2) dx$$

Step 2: Now integrate both sides.

$$\int y dy = \int (3x^2 + 2) dx$$

$$\frac{y^2}{2} = x^3 + 2x + c$$

Notice how we only use one constant. All we have to do now is solve for C . We do this by Plugging In 2 for x and 4 for y .

$$\frac{16}{2} = 2^3 + 4 + C$$

$$C = -4$$

So we can rewrite the equation as $\frac{y^2}{2} = x^3 + 2x - 4$.

Step 3: Now if we Plug In 3 for x , we will get y .

$$\frac{y^2}{2} = 27 + 6 - 4$$

$$y^2 = 58$$

$$y = \pm\sqrt{58}$$

The answer is (E).

PROBLEM 25. $\int \frac{dx}{9+x^2} =$

This is another Inverse Trigonometric Integral.

Step 1: We know that $\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$.

(See problem 9 if you're not sure of this.) The trick here is to get the denominator of the fraction in the integrand to be of the correct form. If we factor 9 out of the denominator we get:

$$\int \frac{dx}{9+x^2} = \int \frac{dx}{9\left(1+\frac{x^2}{9}\right)} = \frac{1}{9} \int \frac{dx}{1+\frac{x^2}{9}} = \frac{1}{9} \int \frac{dx}{1+\left(\frac{x}{3}\right)^2}$$

Step 2: Now if we use u -substitution we will be able to evaluate this integral.

Let $u = \frac{x}{3}$ and $du = \frac{1}{3}dx$ or $3du = dx$. Then we have:

$$\frac{1}{9} \int \frac{dx}{1+\left(\frac{x}{3}\right)^2} = \frac{1}{9} \int \frac{3du}{1+u^2} = \frac{1}{3} \int \frac{du}{1+u^2} = \frac{1}{3} \tan^{-1}(u) + C.$$

Step 3: Now all we have to do is reverse the u -substitution and we're done.

$$\frac{1}{3} \tan^{-1}(u) + C = \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

The answer is (B).

PROBLEM 26. If $f(x) = \cos^3(x+1)$ then $f'(\pi) =$

Think of $\cos^3(x+1)$ as $[\cos(x+1)]^3$.

Step 1: First, we take the derivative of the outside function and ignore the inside functions. The derivative of u^3 is $3u^2$.

We get: $\frac{d}{dx}[u]^3 = 3[u]^2$.

Step 2: Next, we take the derivative of the cosine term and multiply. The derivative of $\cos u$ is $-\sin u$.

$$\frac{d}{dx}[\cos(u)]^3 = -3[\cos(u)]^2 \sin(u)$$

Step 3: Finally, we take the derivative of $x+1$ and multiply. The derivative of $x+1$ is 1.

$$\frac{d}{dx}[\cos(x+1)]^3 = -3[\cos(x+1)]^2 \sin(x+1)$$

The answer is (A).

Note: If you had trouble with this problem, you should review the section on **The Chain Rule**.

PROBLEM 27. $\int x\sqrt{x+3} dx =$

We can do this integral with u -substitution.

Step 1: Let $u = x+3$. Then $du = dx$ and $u-3 = x$.

Step 2: Substituting, we get:

$$\int x\sqrt{x+3} dx = \int (u-3)u^{\frac{1}{2}} du$$

Why is this better than the original integral, you might ask? Because now we can distribute and the integral becomes easy.

Step 3: When we distribute, we get:

$$\int (u-3)u^{\frac{1}{2}} du = \int \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}} \right) du$$

Step 4: Now we can integrate:

$$\int \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}} \right) du = \frac{2}{5} u^{\frac{5}{2}} - 3 \cdot \frac{2}{3} u^{\frac{3}{2}} + C$$

Step 5: Substituting back, we get:

$$\frac{2}{5} u^{\frac{5}{2}} - 3 \cdot \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{5} (x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$$

The answer is (C).

PROBLEM 28. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$

Here, we use the chain rule.

Step 1: First, take the derivative of the outside function.

The derivative of $\ln u$ is $\frac{du}{u}$.

We get:

$$\frac{d}{dx} \ln(\ln(\)) = \frac{1}{\ln(\)}$$

Step 2: Now we take the derivative of the function in the denominator. Once again, the function is $\ln u$.

We get:

$$\frac{d}{dx} \ln(\ln(1-x)) = \frac{1}{\ln(1-x)} \cdot \frac{-1}{1-x} = -\frac{1}{(1-x)\ln(1-x)}$$

The answer is (D).

PROBLEM 29. $\int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{-\frac{\pi}{4}}^0 \cos x \, dx =$

This is a pair of basic Trigonometric Integrals. You should have memorized several trigonometric integrals, particularly $\int \sin x \, dx = -\cos x + C$ and

$$\int \cos x \, dx = \sin x + C$$

Step 1: $\int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{-\frac{\pi}{4}}^0 \cos x \, dx = -\cos x \Big|_0^{\frac{\pi}{4}} + \sin x \Big|_{-\frac{\pi}{4}}^0$